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Loop-Corrected Compactifications of the Heterotic String with Line Bundles

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Abstract

We consider the $E_8 \times E_8$ heterotic string theory compactified on Calabi-Yau manifolds with bundles containing abelian factors in their structure group. Generic low energy consequences such as the generalised Green-Schwarz mechanism for the multiple anomalous abelian gauge groups are studied. We also compute the holomorphic gauge couplings and induced Fayet-Iliopoulos terms up to one-loop order, where the latter are interpreted as stringy one-loop corrections to the Donaldson-Uhlenbeck-Yau condition. Such models generically have frozen combinations of Kähler and dilaton moduli. We study concrete bundles with structure group $SU(N) \times U(1)^M$ yielding quasi-realistic gauge groups with chiral matter given by certain bundle cohomology classes. We also provide a number of explicit tadpole free examples of bundles defined by exact sequences of sums of line bundles over complete intersection Calabi-Yau spaces. This includes one example with precisely the Standard Model gauge symmetry.

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1 Introduction

Over the years string model building techniques have been developed in various corners of the M-theory moduli space. Since one is interested in having gauge symmetry as a general feature, in particular Type I and heterotic constructions have been pursued intensively. However, between these two different constructions there exists a certain asymmetry. For Type I like constructions much effort has gone into the study of so-called intersecting D-brane models, where Standard Model like features could be engineered geometrically (see [1–5] for reviews). It is known that in a mirror symmetric picture, those most simple constructions can be described by magnetised D-branes or by turning on abelian gauge fields on the

world-volume of D9-branes. On the heterotic side, similar constructions, so-called $(0,2)$ models, have been investigated, where in most cases people have considered non-abelian $SU(N)$ bundles embedded into $E_8 \times E_8$ on Calabi-Yau spaces. Various mathematical constructions for such bundles have been applied, such as the monad construction occurring naturally in the framework of $(0,2)$ linear sigma models [6], the spectral cover construction [7] for elliptically fibered Calabi-Yau manifolds as they appear in studying F-theory duality and more recently the method of bundle extensions [8–10] leading to some MSSM like examples [11,12] (see also [13–15]). Such $SU(N)$ bundles allow for breaking the observable gauge symmetry to GUT groups like E_6, $SO(10)$ or $SU(5)$. These gauge symmetries can then be broken further down to the Standard Model group by using Wilson lines. Thus for concrete model building, on the Type I side one is invoking solely $U(1)$ bundles, whereas on the heterotic side mainly $SU(N)$ bundles have been used.

In this article we carry out a more systematic study of model building prospects using $U(1)$ bundles in the $E_8 \times E_8$ heterotic string [1]. As for magnetised D-brane models, one cannot only break the gauge symmetry by Wilson lines (trivial line bundles) but also by turning on non-trivial (internal) abelian gauge fields. The use of these abelian bundles provides much more model building flexibility and implies some new features of the resulting models.

In general, more than one of the $U(1)$ gauge symmetries also contained in the structure group of the background gauge bundle is anomalous. We will show in detail that these anomalies are cancelled by a generalised Green-Schwarz mechanism invoking not only the universal axio-dilaton multiplet but also the internal axio-Kähler multiplets. Some linear combinations of the latter become the longitudinal modes of the anomalous $U(1)$s rendering these gauge fields massive. Thus, in contrast to Wilson line breaking, for the breaking via non-trivial abelian fluxes the $U(1)$ gauge symmetries can become massive. Supersymmetry dictates that there must also arise Fayet-Iliopolous terms, which in their supersymmetric minimum give masses to linear combinations of the dilaton and the Kähler moduli. In fact, we will see that the dilaton part of the FI-terms can be interpreted as a one-loop correction to the tree-level supersymmetry condition, which is nothing else than the Donaldson-Uhlenbeck-Yau condition. We will provide further evidence for this picture from heterotic-Type I duality. Let us emphasize again that in contrast to earlier claims, $U(1)$ bundles actually freeze combinations of the axio-dilaton multiplet and the h_{11} axio-Kähler multiplets. As a consequence, such models do not necessarily impose the condition $h_{11} > 1$.

We also compute the one-loop corrected holomorphic gauge kinetic functions for both the non-abelian and the abelian gauge factors. Including the one-loop

[1] A study of $U(N)$ bundles in the framework of the spectral cover construction has appeared recently in [16]. Besides that, the only constructions known to us are some scattered results on aspects of four dimensional models [17,18] and a few papers on six-dimensional models [19–22].

threshold corrections, one finds that generically the gauge couplings for the $U(1)$s are all different. Therefore, as expected from our original motivation, these heterotic string models with line bundles show completely analogous features as their corresponding Type I counterparts. This might help to resolve some of the puzzles in the literature about the different structure of heterotic and Type I compactifications [23].

This paper is organised as follows: In section 2 we summarize the general construction of heterotic string compactifications involving also $U(N)$ bundles on Calabi-Yau manifolds using (exact) sequences and discuss the generalized Green-Schwarz mechanism cancelling the various $U(1)$ gauge anomalies. Even though there exists an extensive literature on this subject, for completeness we also discuss the holomorphic gauge kinetic functions. As one of the main issues of this paper, we address the generation of Fayet-Iliopoulus terms for the $U(1)$ symmetries, which together with Type I-heterotic duality provide striking evidence for the existence of a stringy one-loop correction to the Donaldson-Uhlenbeck-Yau stability condition. In section 3 we investigate bundles with structure group $SU(4)\times U(1)$, which give rise to GUT-like models in four dimensions with gauge symmetry $SU(5)$. To show that such vacua really exist, we provide a concrete example of such bundles both on the Quintic and on a complete intersection Calabi-Yau (CICY). Sections 4 and 5 are devoted to more general bundles having two and three $U(1)$ factors in their structure group, for which we study the various gauge symmetry enhancements and the possibility of embedding the MSSM in such a model. Indeed we exemplify that one can find concrete bundles which lead to just the Standard Model gauge symmetry (in addition to a hidden E_8 gauge symmetry). Section 6 contains our conclusions. Some technical details are displayed in appendix A - C.

2 Calabi-Yau manifolds with $U(N)$ bundles

We consider the heterotic string compactified on a Calabi-Yau manifold $\mathcal{M}$ endowed with an additional vector bundle whose structure group is embedded into the $SO(32)$ or $E_8 \times E_8$ ten-dimensional gauge group. In this paper, for phenomenological reasons, we will be concerned with the $E_8 \times E_8$ heterotic string. In the following the notation is adjusted to this case.

2.1 String model building constraints

Up to now, most papers on heterotic compactifications have considered only bundles with structure group $SU(N)$. In this article we investigate vector bundles

of the following form

$$W = \bigoplus_{i=1}^{K} V_{n_i} \oplus \bigoplus_{m=1}^{M} L_m, \tag{1}$$

where the V_{n_i} are $SU(n_i)$ or $U(n_i)$ bundles and the L_m denote some complex line bundles with structure group $U(1)$ each. As is well known, to leading order in α' the string equations of motion respectively the supersymmetry conditions put several constraints on the vector bundle W which can live on the Calabi-Yau. It is one of the main results of this paper that beyond tree level there arise additional important constraints at one-loop level which modify the picture in such a way as to make it consistent with the dual Type I string constructions. In the following we move slightly ahead and summarize the main model building rules arising at string tree and one-loop level.

- The vector bundle W has to admit spinors, which means that the obstruction given by the second Stiefel-Whitney class has to vanish, i.e.

$$c_1(W) \in H^2(\mathcal{M}, 2\mathbb{Z}). \tag{2}$$

- At string tree level, the connection of the vector bundle has to satisfy the well-known zero-slope limit of the Hermitian Yang-Mills equations,

$$F_{ab} = F_{\overline{a}\overline{b}} = 0, \quad g^{a\overline{b}}\, F_{a\overline{b}} = 0. \tag{3}$$

  The first equation implies that each term in (1) has to be a *holomorphic* vector bundle. Due to its holomorphicity, this constraint can only arise as an F-term in the effective $\mathcal{N} = 1$ supergravity description and therefore does not receive any perturbative corrections in α' or the string loop expansion [24]. The second equation in (3) is actually the special case of the general hermitian Yang-Mills equation

$$J \wedge J \wedge F = \mu(W)\, \mathrm{vol}_{\mathcal{M}}\, I, \tag{4}$$

  where $\mathrm{vol}_{\mathcal{M}}$ is the volume form on the base manifold of the bundle normalized such that $\int_{\mathcal{M}} \mathrm{vol}_{\mathcal{M}} = 1$, and I refers to the identity matrix acting on the fibre. Recall that the slope μ of a coherent sheaf $\mathcal{V}$ with respect to a Kähler form J on a manifold $\mathcal{M}$ is defined as

$$\mu(\mathcal{V}) = \frac{1}{\mathrm{rk}(\mathcal{V})} \int_{\mathcal{M}} J \wedge J \wedge c_1(\mathcal{V}). \tag{5}$$

  According to a theorem by Uhlenbeck-Yau, (4) has a unique solution if and only if the vector bundle W in question is μ-stable, i.e. if for each coherent subsheaf $\mathcal{V}$ of W with $0 < \mathrm{rk}(\mathcal{V}) < \mathrm{rk}(W)$ one has

$$\mu(\mathcal{V}) < \mu(W). \tag{6}$$

Consequently, the zero-slope limit of the hermitian Yang-Mills equations (3) relevant at tree level is satisfied precisely by holomorphic μ-stable bundles which meet in addition the integrability condition

$$\int_{\mathcal{M}} J \wedge J \wedge c_1(V_{n_i}) = 0, \qquad \int_{\mathcal{M}} J \wedge J \wedge c_1(L_m) = 0, \tag{7}$$

to be satisfied for all n_i, m. We will refer to the latter constraints in the following as the tree-level Donaldson-Uhlenbeck-Yau (DUY) equation. At string tree level the DUY condition imposes a set of constraints on the Kähler moduli. However, we will see in the course of this paper that the integrability condition (7) arises from a D-term in the effective supergravity description, more concretely it is the string tree level Fayet-Iliopolous term. Consequently, it is actually a specific combination of the Kähler moduli and the $U(1)$ charged matter fields which will be frozen by the tree level requirement of supersymmetry. We will however not make the appearance of these matter fields explicit in the remainder of this paper.

- As mentioned, the DUY condition plays the role of a D-term constraint and as such can, at the perturbative level, in principle be subject to at most one-loop corrections [25]. We will indeed demonstrate the presence of such a one-loop correction to the DUY equation

$$\begin{aligned} &\int_{\mathcal{M}} J \wedge J \wedge c_1(L_n) - \\ &\frac{1}{2}\, \ell_s^4\, g_s^2 \int_{\mathcal{M}} c_1(L_n) \wedge \left(\sum_{i=1}^{K} \mathrm{ch}_2(V_{n_i}) + \sum_{m=1}^{M} a_m\, c_1^2(L_m) + \frac{1}{2}\, c_2(T) \right) = 0, \end{aligned} \tag{8}$$

where $g_s = e^{\phi_{10}}$, $\ell_s = 2\pi\sqrt{\alpha'}$ and the coefficients $a_m \in \mathbb{Z}/2$ depend on the concrete embedding of the $U(1)$ structure groups into $E_8 \times E_8$ [2]. Here, the bracket in the second line of (8) contains only sums over those bundles which sit in the same E_8 factor as the line bundle L_n. Clearly for $SU(N)$ bundles the one-loop correction vanishes just as the tree level constraint and one only gets a non-trivial condition for $U(N)$ gauge bundles. The one-loop correction implies that actually combinations of Kähler moduli and the dilaton are frozen by the supersymmetry condition. More precisely, if the $c_1(V_{n_i}), c_1(L_m) \in H^2(\mathcal{M}, \mathbb{Z})$ span a D dimensional subspace of $H^2(\mathcal{M}, \mathbb{Z})$, D combinations of the Kähler moduli and the dilaton become massive. Observing a modified integrability condition for $U(N)$ bundles, there should exist a corresponding one-loop correction to the hermitian Yang-Mills equation and to the μ-stability condition.

[2]Naively one might have thought that always $a_m = \frac{1}{2}$, but this is not true. Indeed, for the models with $SU(n) \times U(1)^M$ bundles, we find that $a_m = 6, 10, 15$ if the $U(1)_m$ factor arises via the breaking of the subgroups $E_6 \to SO(10) \times U(1)$, $SO(10) \to SU(5) \times U(1)$ and $SU(5) \to SU(3) \times SU(2) \times U(1)$, respectively, as we will show in the following sections.

- The Bianchi identity for the three-form $H = dB - \frac{\alpha'}{4}(\omega_Y - \omega_L)$,

$$0 = dH = \mathrm{tr}(F_1^2) + \mathrm{tr}(F_2^2) - \mathrm{tr}(R^2), \tag{9}$$

imposes the so-called tadpole condition for the background bundles. Here we explicitly distinguish between the observable and hidden gauge sectors and the trace is over the fundamental representation of E_8 for the gauge bundles and over the fundamental representation of $SO(1,9)$ for the curvature two-form. Often one also uses the symbol $\mathrm{Tr}(F_i) = 30\,\mathrm{tr}(F_i)$ to formally distinguish between traces in the adjoint and fundamental representation of E_8. For direct sums of $SU(N)$ bundles the resulting tadpole cancellation condition takes the familiar form

$$\sum_i c_2(V_{n_i}) = c_2(T), \tag{10}$$

where T is the tangent bundle of the Calabi-Yau manifold. There can be additional non-perturbative contributions to (10) from wrapped 5-branes, but we are not considering them in this paper [26–28]. Including abelian bundles, the resulting topological condition can generally be written as

$$\sum_{i=1}^{K} \mathrm{ch}_2(V_{n_i}) + \sum_{m=1}^{M} a_m\, c_1^2(L_m) = -c_2(T). \tag{11}$$

Note that the spinor condition (2) guarantees that the left hand side takes values in $H^4(\mathcal{M}, \mathbb{Z})$.

- In order to finally get a well defined four-dimensional theory, one has to ensure that the structure group G of the bundle W can be embedded into $E_8 \times E_8$. The observable gauge group in four dimensions H is the commutant of the structure group G of W in $E_8 \times E_8$. It is clear that the structure group of all line bundles $U(1)^M$ is contained in H ($U(1)$ factors of type (i) according to [17, 29, 30]), but there might be additional $U(1)$ factors in H not contained in the structure group ($U(1)$ factors of type (ii)).

Since D combinations of the dilaton and the Kähler moduli become massive, supersymmetry implies that the same must happen to their axionic superpartners as well. In fact, as we will discuss, D of these axions mix with D of the $U(1)$ gauge bosons making them massive, so that the final gauge symmetry in four dimensions is reduced by these D abelian $U(1)$ factors. These massive $U(1)$s nevertheless survive as perturbative global symmetries providing for instance selection rules for correlation functions.

The massless spectrum is determined by various cohomology classes

$$H^*(\mathcal{M}, \bigotimes_{i=1}^{K} \bigwedge^{p_i} V_{n_i} \otimes \bigotimes_{m=1}^{M} L_m^{q_m}), \tag{12}$$

where the charges p_i and q_m can be derived from the explicit embedding of the structure group into $E_8 \times E_8$. The net-number of chiral matter multiplets is given by the Euler characteristic of the respective bundle $\mathcal{W}$ in (12), which can be computed using the Riemann-Roch-Hirzebruch theorem

$$\begin{aligned}\chi(\mathcal{M},\mathcal{W}) &= \sum_{i=0}^{3}(-1)^i \dim(H^i(\mathcal{M},\mathcal{W})) \\ &= \int_{\mathcal{M}} \left[\mathrm{ch}_3(\mathcal{W}) + \frac{1}{12}\, c_2(T)\, c_1(\mathcal{W})\right]. \end{aligned} \tag{13}$$

Of course, allowing this general structure of bundles gives rise to a plethora of new model building possibilities within the heterotic framework. In the following we will discuss some new ways of how Standard-like models can arise in this setting. To start with, we focus in this paper on three concrete bundle types of the form $V_4 \oplus \bigoplus_{m=1}^M L_m$ and $V_3 \oplus \bigoplus_{m=1}^M L_m$ for up to three $U(1)$ factors.

2.2 Vector bundles via exact sequences of line bundles

In order to construct explicit models one needs a concrete description of vector bundles on Calabi-Yau spaces. There are various constructions known in the literature. For elliptically fibered Calabi-Yau spaces one might want to apply the spectral cover construction of Freedman-Morgan-Witten [7] or the method of bundle extensions [8–10]. In this paper we will however use the definition of vector bundles as they naturally appear in the $(0,2)$ generalization of the linear sigma model. Here they are given by (exact) sequences of direct sums of line bundles on the Calabi-Yau.

Assume that we have a Calabi-Yau manifold $\mathcal{M}$ which is given by a complete intersection in some toric variety and has $k = h_{11}$ Kähler parameters. A line bundle L on $\mathcal{M}$ is specified completely by its first Chern class which takes values in $H^2(\mathcal{M},\mathbb{Z})$ and can be expanded as

$$c_1(L) = \sum_{i=1}^{h_{11}} n_i\, \omega_i, \tag{14}$$

where the ω_i form a basis of $H^2(\mathcal{M},\mathbb{Z})$ and $n_i \in \mathbb{Z}$. One also denotes such a line bundle as $\mathcal{O}(n_1,\ldots,n_k)$. Then a vector bundle V of rank r is defined by the cohomology of the monad

$$0 \to \; \mathcal{O}|_{\mathcal{M}}^{\oplus p} \xrightarrow{g} \bigoplus_{a=1}^{r+p+1} \mathcal{O}(n_{a,1},\ldots,n_{a,k})|_{\mathcal{M}} \xrightarrow{f} \mathcal{O}(m_1,\ldots,m_k)|_{\mathcal{M}} \to 0, \tag{15}$$

i.e. V =Kern(f)/Im(g) and $p \geq 0$. Such a sequence can be split into two exact

sequences

$$
\begin{aligned}
&0\longrightarrow\mathcal{O}|_{\mathcal{M}}^{\oplus p}\longrightarrow\bigoplus_{a=1}^{r+p+1}\mathcal{O}(n_{a,1},\dots,n_{a,k})|_{\mathcal{M}}\longrightarrow\mathcal{E}|_{\mathcal{M}}\longrightarrow 0,\\
&0\longrightarrow V|_{\mathcal{M}}\longrightarrow\mathcal{E}|_{\mathcal{M}}\longrightarrow\mathcal{O}(m_1,\dots,m_k)|_{\mathcal{M}}\longrightarrow 0.
\end{aligned}
\tag{16}
$$

Of course one has to ensure that the maps g and f are such that the sequences really define a bona-fide vector bundle and not only a non-locally free coherent sheaf.

The total Chern class of the vector bundle V is then given by

$$
c(V)=\frac{\prod_{a=1}^{r+p+1}\left(1+\sum_i n_{a,i}\,\omega_i\right)}{\left(1+\sum_i m_i\,\omega_i\right)},
\tag{17}
$$

which implies in particular that the first Chern class of V is

$$
c_1(V)=\sum_{i=1}^{h_{11}}\left(\sum_{a=1}^{r+p+1}n_{a,i}-m_i\right)\omega_i.
\tag{18}
$$

Clearly, for $SU(N)$ bundles one has $c_1(V)=0$, whereas for $U(N)$ bundles at first sight no condition on the first Chern classes arises. Using the Chern classes one can compute the chiral massless spectrum by means of (13). Often also the bundle $\bigwedge^2 V$ appears, whose Chern classes can be determined using [31]

$$
\begin{aligned}
\mathrm{ch}_1(\textstyle\bigwedge^2 V)&=(r-1)\,\mathrm{ch}_1(V),\\
\mathrm{ch}_2(\textstyle\bigwedge^2 V)&=(r-2)\,\mathrm{ch}_2(V)+\frac{1}{2}\,\mathrm{ch}_1^2(V),\\
\mathrm{ch}_3(\textstyle\bigwedge^2 V)&=(r-4)\,\mathrm{ch}_3(V)+\mathrm{ch}_2(V)\,\mathrm{ch}_1(V).
\end{aligned}
\tag{19}
$$

In order to arrive at the complete (non-chiral) massless spectrum one really has to compute the cohomology classes $H^i(\mathcal{M},\mathcal{W})$. The methods to compute them are known in the literature [17,32] and we summarize part of them in Appendix A.

We pointed out already that there are in principle two distinct ways of embedding abelian groups into E_8. Either one chooses V_{n_i} to have structure group $SU(n_i)$. In that case the group theoretic $U(1)$-charges of the states upon decomposition of E_8 directly give us the powers of the respective line bundles in the cohomology class (12) counting their multiplicities. Clearly, the various line bundles are not correlated among each other and in particular V_{n_i} gives no contribution to the $U(1)$-charges. We will exemplify this class of constructions by taking the structure group of V to be $SU(4)$ with one and two additional line bundles in sections 3 and 4, respectively, and by choosing an $SU(3)$ bundle with three line bundles in section 5.

Alternatively, one can embed $U(N)$ bundles into E_8 by means of a particular construction where one actually starts with a $U(N)\times U(1)^M$ bundle with $c_1(W)=0$. Throughout the remainder of this article, in contrast to the ansatz (1) for $SU(N)\times U(1)^M$ bundles, we adopt the notation

$$W=\bigoplus_{i=1}^{K} V_{n_i}\oplus\bigoplus_{m=1}^{M} L_m^{-1} \tag{20}$$

for $U(N)\times U(1)^M$ bundles. There is a natural way of getting the defining line bundle data of such bundles from the one of an $SU(N+M)$ bundle. Say we have found already an $SU(N+M)$ bundle W or rather a coherent sheaf, as for the coming construction it is not essential that W is locally free. Then we can split off M of the line bundles and define

$$L_m^{-1}=\mathcal{O}(n_{m,1},\ldots,n_{m,k})|_{\mathcal{M}} \tag{21}$$

for $m=1,\ldots,M$. The remaining sequence now reads

$$0\to\ \mathcal{O}|_{\mathcal{M}}^{\oplus p}\xrightarrow{g}\bigoplus_{a=M+1}^{r+p+1}\mathcal{O}(n_{a,1},\ldots,n_{a,k})|_{\mathcal{M}}\xrightarrow{f}\mathcal{O}(m_1,\ldots,m_k)|_{\mathcal{M}}\to 0 \tag{22}$$

and defines a $U(N)$ bundle V if the maps f and g can be suitably chosen. Clearly these bundles satisfy $c_1(V)=\sum_{m=1}^{M}c_1(L_m)$ and, using (17), one can show that they obey the constraint

$$\chi(V)+\sum_{m=1}^{M}\chi(L_m^{-1})=\chi(W). \tag{23}$$

In contrast to what we said about $SU(N)$ bundles, now the $U(1)$ charges of the states comprise contributions from both the $U(N)$ bundle V and the line bundles, which after all are not independent but are chosen just to absorb the diagonal $U(1)$-charge of $U(N)$ in the splitting $SU(N+M)\to U(N)\times U(1)^M$. Now one has to fix the embedding of the $U(1)^M$ part of the structure group into E_8 respectively $SU(N+M)$. For $i=1,\ldots,M$ this can be described by the charges

$$Q_i=(\underbrace{Q_i(V),\ldots,Q_i(V)}_{N\ \text{times}},Q_i(L_1^{-1}),\ldots,Q_i(L_M^{-1})) \tag{24}$$

with

$$N\,Q_i(V)+\sum_{m=1}^{M}Q_i(L_m^{-1})=0. \tag{25}$$

For the detailed computation of the various anomalies associated with the $U(1)$-factors, it will turn out to be convenient to introduce the matrix

$$\mathcal{Q}_{im} = Q_i(V) + Q_i(L_m). \tag{26}$$

Again, we will make this construction more explicit for a $U(4)\times U(1)$ and $U(4)\times U(1)^2$ bundle as well as a model involving $U(3)\times U(1)^3$ in sections 3-5.

2.3 $U(1)$ gauge factors and the Green-Schwarz mechanism

As usual in string theory, whereas all irreducible anomalies cancel directly due to the string consistency constraints [33] such as tadpole cancellation, the factorisable ones do not. In four dimensions that means that all non-abelian cubic gauge anomalies do cancel, whereas the mixed abelian-nonabelian, the mixed abelian-gravitational and the cubic abelian ones do not. As we will discuss in this section, they have to be cancelled by a generalised Green-Schwarz mechanism [3]. Since each $U(1)$ bundle in the structure group of the bundle implies a $U(1)$ gauge symmetry in four dimensions, all these latter three anomalies appear. We restrict ourselves for brevity to the case that V has structure group $SU(N)$; we will indicate the modifications in the otherwise largely analogous analysis of $U(N)$ bundles at the end of this section.

Let us write the ten-dimensional gauge fields F^{10} as $F^{10}_i = F_i + \overline{F}_i$, where F_i is the external four dimensional part taking values in H and $\overline{F}_i$ denotes the internal six-dimensional part, which takes values in the structure group G of the bundle. Recall that the $U(1)$ factors of type (i) are special as they appear both in G and H. Computing the field theory mixed $U(1)_m$-$SU(N)^2$ and mixed $U(1)_m$-$G^2_{\mu\nu}$ anomalies for $m\in\{1,\dots,M\}$, one finds that their anomaly six-forms are of the universal form

$$\begin{aligned} A_{U(1)_m - SU(N)^2} \;&\sim\; f_m \wedge \mathrm{tr}F_1^2 \;\left[\int_{\mathcal{M}} \overline{f}^m \wedge \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right)\right], && (27)\\ A_{U(1)_m - G^2_{\mu\nu}} \;&\sim\; f_m \wedge \mathrm{tr}R^2 \;\left[\int_{\mathcal{M}} \overline{f}^m \wedge \left(12\,\mathrm{tr}\overline{F}_1^2 - 5\,\mathrm{tr}\overline{R}^2\right)\right]. && (28)\end{aligned}$$

Here we have denoted the four-dimensional $U(1)$ two-form field strengths as f_m and the internal ones as $\overline{f}^m$. Moreover, we are here considering only line bundles in the first E_8 factor, the story for the second one being completely analogous. The $U(1)_m$-$U(1)_n$-$U(1)_p$ anomalies are slightly more complicated and can be written in the following general form

$$A_{U(1)_m - U(1)_n - U(1)_p} \;\sim\; f_m \wedge f_n \wedge f_p \left[\int_{\mathcal{M}} \overline{f}^m \wedge \delta_{np}\left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right) + \right.$$

[3] The Green-Schwarz mechanism for several $U(1)$ symmetries in $E_8\times E_8$ heterotic compactifications has also been discussed in detail in [30], where however the authors have come to somehow different conclusions.

$$c_{mnp}\, \overline{f}^m \wedge \overline{f}^n \wedge \overline{f}^p \Big]. \tag{29}$$

Here we have assumed that for at least two $U(1)$s being identical, the single one is $U(1)_m$. For $m \neq n \neq p$ the first term in (29) is absent. For $n = p$ the relative factor between the first and the second term in (29) can be expressed as

$$c_{mnn} = \frac{2}{3}\, \mathrm{tr}_{E_8}(Q_n^2)\, \sigma_{mnn}, \tag{30}$$

where Q_n is the generator of $U(1)_n$, whose trace is related to the coefficient a_n in (11) via $4\, a_n = \mathrm{tr}_{E_8}(Q_n^2)$. The σ_{mnn} denotes the symmetry factor of the anomalous diagram, i.e. $\sigma_{mmm} = 1$ and $\sigma_{mnn} = 3$ for $m \neq n$.

Let us demonstrate that these anomalies are cancelled by a generalized Green-Schwarz mechanism [34] [4]. Throughout this paper we are working in string frame. In ten dimensions the gauge anomalies are cancelled by the counter term [36]

$$S_{GS} = \frac{1}{48\,(2\pi)^5\,\alpha'} \int B \wedge X_8, \tag{31}$$

where B is the string two-form field and the eight-form X_8 reads, as usual,

$$X_8 = \frac{1}{24}\mathrm{Tr}F^4 - \frac{1}{7200}\left(\mathrm{Tr}F^2\right)^2 - \frac{1}{240}\left(\mathrm{Tr}F^2\right)\left(\mathrm{tr}R^2\right) + \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}\left(\mathrm{tr}R^2\right)^2. \tag{32}$$

Explicitly taking care of the two E_8 factors by writing $F = F_1 + F_2$ one gets

$$\begin{aligned} X_8 \;=\; & \frac{1}{4}\left(\mathrm{tr}F_1^2\right)^2 + \frac{1}{4}\left(\mathrm{tr}F_2^2\right)^2 - \frac{1}{4}\left(\mathrm{tr}F_1^2\right)\left(\mathrm{tr}F_2^2\right) - \frac{1}{8}\left(\mathrm{tr}F_1^2 + \mathrm{tr}F_2^2\right)\left(\mathrm{tr}R^2\right) + \\ & \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}\left(\mathrm{tr}R^2\right)^2. \end{aligned} \tag{33}$$

Using the tadpole cancellation condition (9), we dimensionally reduce this term to

$$\begin{aligned} S_{GS} \;=\; & \frac{1}{64\,(2\pi)^5\,\alpha'} \int B \wedge \left(\mathrm{tr}F_1^2\right)\left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right) && (34)\\ & - \frac{1}{768\,(2\pi)^5\,\alpha'} \int B \wedge \left(\mathrm{tr}R^2\right)\left(\mathrm{tr}\overline{R}^2\right) && (35)\\ & + \frac{1}{48\,(2\pi)^5\,\alpha'} \int B \wedge \left[\mathrm{tr}(F_1\overline{F}_1)\right]^2 && (36)\\ & + \frac{1}{32\,(2\pi)^5\,\alpha'} \int B \wedge \mathrm{tr}(F_1\overline{F}_1)\left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right), && (37) \end{aligned}$$

where we have shown only the terms for the first E_8. There are exactly the same terms (34),(36),(37) for the second E_8 by replacing $F_1 \to F_2$ and $\overline{F}_1 \to \overline{F}_2$

[4] Many useful formulas on group theoretical identities can be found in [35].

as well as a mixed term involving $\mathrm{tr}(F_1\overline{F}_1)\,\mathrm{tr}(F_2\overline{F}_2)$. Note, however, that our models in this article only involve line bundles in one of the two E_8-factors. For concreteness let us discuss the $U(1)$-$SU(N)^2$ and $U(1)$-$G^2_{\mu\nu}$ anomalies in detail. It is convenient to make use of a basis of two-forms ω_k, $k = 1, \ldots, h_{11}$ as in section 2.2 and their Hodge dual four-forms $\widehat{\omega}^k$, i.e. they satisfy

$$\int_{\mathcal{M}} \omega_k \wedge \widehat{\omega}_{k'} = \delta_{kk'}. \tag{38}$$

In terms of the string length $\ell_s = 2\pi\sqrt{\alpha'}$ we now expand

$$\begin{aligned}
B^{(2)} &= b_0^{(2)} + \ell_s^2 \sum_{k=1}^{h_{11}} b_k^{(0)}\, \omega_k, \quad \mathrm{tr}\overline{F}_1^2 = (2\pi)^2 \sum_{k=1}^{h_{11}} (\mathrm{tr}\overline{F}_1^2)_k\, \widehat{\omega}_k, \\
\overline{f}^m &= 2\pi \sum_{k=1}^{h_{11}} \overline{f}_k^m\, \omega_k, \qquad\quad \mathrm{tr}\overline{R}^2 = (2\pi)^2 \sum_{k=1}^{h_{11}} (\mathrm{tr}\overline{R}^2)_k\, \widehat{\omega}_k,
\end{aligned} \tag{39}$$

where for dimensional reasons we have introduced appropriate powers of α'. Note that $\overline{f}_k^m \in \mathbb{Z}$. Inserting these expansions into the GS-terms (34) and (35) gives rise to GS-terms in four dimensions

$$\begin{aligned}
S_{GS} &= \frac{1}{64\,(2\pi)} \int_{\mathbf{R}_{1,3}} \sum_{k=1}^{h_{11}} \left(b_k^{(0)}\, \mathrm{tr}F_1^2 \right)\; (\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2)_k \qquad (40) \\
&\quad - \frac{1}{768\,(2\pi)} \int_{\mathbf{R}_{1,3}} \sum_{k=1}^{h_{11}} \left(b_k^{(0)}\, \mathrm{tr}R^2 \right)\; (\mathrm{tr}\overline{R}^2)_k. \qquad (41)
\end{aligned}$$

As indicated, from the four-dimensional point of view the axions $b_k^{(0)}$ are zero-forms. However, in addition the term (37) gives rise to a mass term for the four-dimensional two-form field $b_0^{(2)}$

$$S^0_{mass} = \frac{1}{32\,(2\pi)^5\alpha'} \int_{\mathbf{R}_{1,3}} \sum_{m=1}^{M} \left(b_0^{(2)} \wedge f_m \right)\, \mathrm{tr}_{E_8}(Q_m^2) \int_{\mathcal{M}} \overline{f}^m \wedge (\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2), \tag{42}$$

where we have assumed that $\mathrm{tr}_{E_8}(Q_m\, Q_n) = 0$ for $m \neq n$, which is indeed satisfied for all $U(1)$ symmetries discussed in this article. This mass term for the universal axion is only present for $U(1)$ symmetries of type (i), reflecting the fact that for the $E_8 \times E_8$ heterotic string $U(1)$ factors of type (ii) are always non-anomalous.

To cancel the anomalies one needs a GS-term for the external axion $b_0^{(2)}$ and mass terms for the internal ones $b_k^{(2)}$. All these terms arise from the following kinetic term in the 10D effective action

$$S_{kin} = -\frac{1}{4\kappa_{10}^2} \int e^{-2\phi_{10}}\, H \wedge \star_{10} H, \tag{43}$$

where $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 (\alpha')^4$ and the heterotic 3-form field strength reads $H = dB^{(2)} - \frac{\alpha'}{4}(\omega_Y - \omega_L)$ involving the gauge and gravitational Chern-Simons terms. Let us denote the dual 6-form of $B^{(2)}$ as $B^{(6)}$, i.e. $\star_{10}\, dB^{(2)} = e^{2\phi_{10}}\, dB^{(6)}$. Then the essential term contained in (43) is

$$S_{\rm kin} \;=\; \frac{\alpha'}{8\kappa_{10}^2} \int (\mathrm{tr} F_1^2 - \mathrm{tr} R^2) \wedge B^{(6)}. \tag{44}$$

We introduce the dimensionally reduced zero- and two-forms

$$B^{(6)} = \ell_s^6\, b_0^{(0)}\, \mathrm{vol}_6 + \ell_s^4 \sum_{k=1}^{h_{11}} b_k^{(2)}\, \widehat{\omega}_k, \tag{45}$$

where vol_6 is the normalized volume form on $\mathcal{M}$, i.e $\int_{\mathcal{M}} \mathrm{vol}_6 = 1$. The two-forms $b_k^{(2)}$ in (45) can be proven to satisfy $\star_4\, db_k^{(2)} = e^{2\phi_{10}}\, db_k^{(0)}$ for all $k \in \{1, \ldots, h_{11}\}$. Then the first term in (45) and equation (44) give first rise to a four-dimensional GS-term

$$S_{GS}^0 = \frac{1}{8\pi} \int_{\mathbf{R}_{1,3}} b_0^{(0)} \wedge (\mathrm{tr} F_1^2 - \mathrm{tr} R^2), \tag{46}$$

where $\star_4\, db_0^{(2)} = e^{2\phi_{10}}\, db_0^{(0)}$. In addition, reducing such that F takes values in the various $U(1)$s with one factor external and the other one internal one finds mass terms for the various internal axions. Performing the dimensional reduction we eventually arrive at four-dimensional couplings of the form

$$S_{\rm mass} = \frac{1}{2\ell_s^2} \int_{\mathbf{R}_{1,3}} \sum_{m=1}^{M} \sum_{k=1}^{h_{11}} \left(f_m \wedge b_k^{(2)} \right)\, \mathrm{tr}_{E_8}(Q_m^2)\, \overline{f}_k^{\,m}. \tag{47}$$

The two GS-couplings (40,46) and two mass terms (42,47) have precisely the right form to generate tree-level graphs of the form shown in Figure 1, which provide couplings of the same type as the ones appearing in the mixed gauge anomalies. For the mixed-abelian non-abelian GS contribution we get

$$A^{GS}_{U(1)_m - SU(N)^2} \sim \frac{\mathrm{tr}_{E_8}(Q_m^2)}{64(2\pi)^6\, \alpha'}\, f_m \wedge \mathrm{tr} F_1^2\, \left[\int_{\mathcal{M}} \overline{f}^m \wedge \left(\mathrm{tr} \overline{F}_1^2 - \frac{1}{2} \mathrm{tr} \overline{R}^2 \right) \right]. \tag{48}$$

For the mixed-abelian-gravitational anomaly the contributions from internal axions and the four-dimensional one are different but they do add precisely up to

$$\begin{aligned} A^{GS}_{U(1)_m - G_{\mu\nu}^2} \;&\sim\; -\frac{\mathrm{tr}_{E_8}(Q_m^2)}{128(2\pi)^6\, \alpha'}\, f_m \wedge \mathrm{tr} R^2\, \left[\int_{\mathcal{M}} \overline{f}^m \wedge \left(\mathrm{tr} \overline{F}_1^2 - \frac{1}{2} \mathrm{tr} \overline{R}^2 \right) \right. \\ &\qquad\qquad \left. + \frac{1}{12} \int_{\mathcal{M}} \overline{f}^m \wedge \left(\mathrm{tr} \overline{R}^2 \right) \right] \\ &=\; -\frac{\mathrm{tr}_{E_8}(Q_m^2)}{128(2\pi)^6\, \alpha'}\, f_m \wedge \mathrm{tr} R^2\, \left[\int_{\mathcal{M}} \overline{f}^m \wedge \left(\mathrm{tr} \overline{F}_1^2 - \frac{5}{12} \mathrm{tr} \overline{R}^2 \right) \right]. \end{aligned} \tag{49}$$

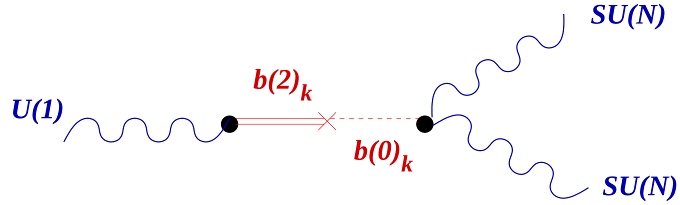

Figure 1: Green-Schwarz counterterm for the mixed gauge anomaly.

Along the same lines, one can also show that the mixed $U(1)^3$ anomalies cancel, where here also the Green-Schwarz couplings (36) contribute.

Finally, let us discuss the mass terms (42,47) in more detail. Independently of the number of anomalous $U(1)$ gauge factors, the number of massive $U(1)$ gauge fields is given by the rank of the matrix

$$\mathrm{M}_{mk} = \begin{cases} \frac{\mathrm{tr}_{E_8}(Q_m^2)}{2\ell_s^2}\, \overline{f}^m_k & \text{for } k \in \{1,\ldots,h_{11}\} \\ \frac{\mathrm{tr}_{E_8}(Q_m^2)}{32\ell_s^2} \int_{\mathcal{M}} \frac{\overline{f}^m}{(2\pi)} \wedge \frac{\left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\right)}{(2\pi)^2} & \text{for } k=0. \end{cases} \tag{50}$$

Let us point out that all mass terms are of the same order in both string and sigma model perturbation theory. The number of massive $U(1)$s will always be at least as big as the number of anomalous $U(1)$s, but as in the Type I case [37], in principle their numbers do not necessarily have to agree. Though all entries in the mass matrix are of order M_s^2, the mass eigenstates of the gauge bosons can have masses significantly lower than the string scale.

Analogously, the number of massive axions $b_i^{(0)}$, $i = 0,\ldots,h_{11}$, is given by the rank of M. Since these axions are just the fields which complexify the Kähler moduli and the dilaton, supersymmetry dictates that the same number of the latter should also get a mass. However, the only source for mass terms so far is the tree-level DUY equation. Expand

$$J = \ell_s^2 \sum_i \alpha_i\, \omega_i \quad \text{and} \quad J\wedge J = \ell_s^4 \sum_i \sigma_i\, \widehat{\omega}_i, \tag{51}$$

where $\sigma_i = \sum_{j,k=1}^{h_{11}} d_{ijk}\alpha_j\alpha_k$ and $d_{ijk} = \int_{\mathcal{M}} \omega_i \wedge \omega_j \wedge \omega_k$ denotes the triple intersection number. Then the tree-level DUY condition can be written as

$$\int_{\mathcal{M}} J\wedge J\wedge c_1(L_m) = \ell_s^4 \sum_{k=1}^{h_{11}} \sigma_k\, \overline{f}^m_k = 0, \tag{52}$$

which tells us that certain linear combinations of the Kähler moduli become massive. However, the dilaton mass terms are missing! Analyzing Fayet-Iliopolous terms, we will see in section 2.5 that there must exist a stringy one-loop correction to the DUY condition.

Let us complete this section by briefly commenting on what happens for the case of $U(N)\times U(1)^M$ bundles. The above discussion applies equally well to these kinds of models with minor changes in the concrete expressions for the various anomalies. Defining for $m = 1, \ldots, M$

$$\widehat{\overline{f}}^m = \sum_{n=1}^{M} \mathcal{Q}_{mn}\, \overline{f}^n \tag{53}$$

in terms of the charge matrix (26), one finds in this case

$$\begin{aligned} A_{U(1)_m-SU(N)^2} &\sim f_m \wedge \mathrm{tr} F_1^2 \left[\int_{\mathcal{M}} \widehat{\overline{f}}^m \wedge \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2 \right)\right], & (54)\\ A_{U(1)_m-G^2_{\mu\nu}} &\sim f_m \wedge \mathrm{tr} R^2 \left[\int_{\mathcal{M}} \widehat{\overline{f}}^m \wedge \left(12\, \mathrm{tr}\overline{F}_1^2 - 5\, \mathrm{tr}\overline{R}^2 \right)\right], & (55)\end{aligned}$$

and for the cubic abelian anomalies

$$\begin{aligned} A_{U(1)_m-U(1)_n-U(1)_p} \sim\ & f_m \wedge f_n \wedge f_p \Big[\int_{\mathcal{M}} \hat{c}_{mnp}\, \widehat{\overline{f}}^m \wedge \delta_{np} \left(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2 \right) + \\ & \widehat{\overline{f}}^m \wedge \widehat{\overline{f}}^n \wedge \widehat{\overline{f}}^p \Big] \end{aligned} \tag{56}$$

with

$$\hat{c}_{mnp} = \frac{3}{8\,\sigma_{mnp}}\, \mathrm{tr}_{E_8}(Q_m^2). \tag{57}$$

Analogously to the $SU(N) \times U(1)^M$ case, these anomalies are cancelled by the generalized Green-Schwarz mechanism, where now the matrix $\mathcal{Q}$ appears in appropriate places.

2.4 Gauge kinetic function

In this section we extract the holomorphic gauge kinetic function f_a for the $SU(N_a)$ gauge symmetries [36, 38–41]. Recall that up to quadratic order the four-dimensional Yang-Mills Lagrangian takes the form

$$\mathcal{L}_{YM} = \frac{1}{4}\,\mathrm{Re}(f_a)\, F \wedge \star F + \frac{1}{4}\,\mathrm{Im}(f_a)\, F \wedge F. \tag{58}$$

Dimensionally reducing the ten-dimensional tree-level term

$$S_{YM}^{(10)} = \frac{1}{2\kappa_{10}^2} \int e^{-2\phi_{10}}\, \frac{\alpha'}{4} \mathrm{tr}(F \wedge \star_{10} F) \tag{59}$$

one obtains

$$S_{YM}^{(4)} = \frac{1}{2\pi} \int_{\mathbb{R}_{1,3}} \frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} e^{-2\phi_{10}}\, \frac{1}{4} \mathrm{tr}(F \wedge \star_4 F). \tag{60}$$

The axionic coupling is contained in (46)

$$S^0_{GS} = \frac{1}{8\pi}\int_{\mathbf{R}_{1,3}} b_0^{(0)} \wedge \mathrm{tr}(F_1 \wedge F_1), \tag{61}$$

so that the tree level gauge kinetic function is simply $f = S$ with the complexified dilaton defined as

$$S \;=\; \frac{1}{2\pi}\left[e^{-2\phi_{10}}\frac{\mathrm{Vol}(\mathcal{M})}{\ell_s^6} + i\, b_0^{(0)}\right]. \tag{62}$$

However, there are the additional axionic couplings (40)

$$S_{GS} = \frac{1}{128\pi}\int_{\mathbf{R}_{1,3}} \sum_{k=1}^{h_{11}} \left(b_k^{(0)}\,\mathrm{tr}(F\wedge F)\right)\;(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2)_k, \tag{63}$$

which are the axionic part of the one-loop threshold corrections to the gauge couplings. We define the complexified Kähler moduli

$$T_k \;=\; \frac{1}{2\pi}\left[-\alpha_k + i b_k^{(0)}\right], \tag{64}$$

where we used the expansion (51). Here α_k is nothing else than the normalized volume of the two-cycle dual to the 2-form ω_k. In this notation the volume of the Calabi-Yau is given by

$$\mathrm{Vol}(\mathcal{M}) = \frac{1}{6}\int_{\mathcal{M}} J\wedge J\wedge J = \frac{\ell_s^6}{6}\sum_{i,j,k} d_{ijk}\,\alpha_i\,\alpha_j\,\alpha_k. \tag{65}$$

Then the one-loop corrected gauge kinetic function for the non-abelian gauge fields can be written as

$$f = S + \frac{1}{16}\sum_k T_k\,(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2)_k. \tag{66}$$

One can perform a similar computation for the gauge couplings of the abelian gauge factors. Here also the GS-term (36) contributes and the final result for the line bundles all embedded into, say, the first E_8 can be cast into the form

$$\begin{aligned} f_{mn} \;=\; & \mathrm{tr}_{E_8}(Q_m^2)\,\delta_{mn}S + \frac{1}{16}\sum_k \mathrm{tr}_{E_8}(Q_m^2)\,T_k\left[\delta_{mn}(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2)_k + \right.\\ & \left. \frac{4}{3}\mathrm{tr}_{E_8}(Q_n^2)\sum_{i,j} d_{ijk}\overline{f}_i^m\overline{f}_j^n\right], \end{aligned} \tag{67}$$

where also non-diagonal couplings appear. The corresponding expression for two abelian factors embedded into different E_8-factors reads

$$f_{m_1 n_2} = -\frac{1}{12}\sum_k \mathrm{tr}_{E_8^1}(Q_{m_1}^2)\,T_k\,\mathrm{tr}_{E_8^2}(Q_{n_2}^2)\sum_{i,j} d_{ijk}\overline{f}_i^{m_1}\overline{f}_j^{n_2}. \tag{68}$$

Note that after including the one-loop corrections, generically the gauge couplings for the $U(1)$ gauge factors are all different. This is just the S-dual feature of the non-universal gauge couplings in the intersecting D-brane set-up [42].

2.5 Fayet-Iliopoulos terms

Since we are dealing with anomalous $U(1)$ gauge factors, there are potential Fayet-Iliopoulos (FI) terms generated [29, 43–47]. Employing the standard supersymmetric field theory formula

$$D_m \frac{\xi_m}{g_m^2} = D_m \frac{\partial \mathcal{K}}{\partial V_m}\Big|_{V=0}, \tag{69}$$

the FI parameters ξ_m can be computed from the Kähler potential $\mathcal{K}$, which in our case takes the following gauge invariant form

$$\begin{aligned} \mathcal{K} \;\; = \;\; & \frac{M_{pl}^2}{8\pi}\Big[-\ln\Big(S+S^* - \sum_m Q_0^m\, V_m\Big) - \ln\Big(\sum_{i,j,k=1}^{h_{11}} \frac{d_{ijk}}{6}\Big(T_i + T_i^* - \sum_m Q_i^m\, V_m\Big) \\ & \Big(T_j + T_j^* - \sum_m Q_j^m\, V_m\Big)\Big(T_k + T_k^* - \sum_m Q_k^m\, V_m\Big)\Big)\Big]. \end{aligned} \tag{70}$$

Here $\frac{M_{pl}^2}{8\pi} = \kappa_{10}^{-2}\, e^{-2\phi_{10}}\, \mathrm{Vol}(\mathcal{M})$ and V_m denotes the vector superfields [5]. The charges Q_k^m can be identified as the couplings in the mass terms (42,47) using the definition

$$S_{mass} = \sum_{m=1}^{M}\sum_{k=0}^{h_{11}} \frac{Q_k^m}{2\pi\alpha'} \int_{\mathbf{R}_{1,3}} f_m \wedge b_k^{(2)}. \tag{71}$$

In view of (69) the FI terms are read off from the expansion of the Kähler potential to linear order in the gauge fields V_m

$$\begin{aligned} \frac{\xi_m}{g_m^2} \;\; = \;\; & -\frac{\pi}{\ell_s^6}\, \mathrm{tr}_{E_8}(Q_m^2)\Big[e^{-2\,\phi_{10}} \frac{1}{2}\int_{\mathcal{M}} J\wedge J\wedge \frac{\overline{f}_m}{2\pi} - \\ & \frac{\ell_s^4}{16}\int_{\mathcal{M}} \frac{\overline{f}_m}{2\pi} \wedge \frac{\Big(\mathrm{tr}\overline{F}_1^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2\Big)}{(2\pi)^2}\Big]. \end{aligned} \tag{72}$$

Apparently, the first term in (72) arises at string tree-level, whereas the second term in (72) is a one-loop term. Therefore we interpret this result as evidence that there exists a one-loop correction to the DUY condition. In contrast to earlier claims that abelian gauge fluxes freeze some linear combinations of the

[5] Taking only one universal Kähler modulus and setting $Q_k^m = 0$, one recovers the familiar result [38, 48].

Kähler moduli, we now realize that actually linear combinations of the dilaton and the Kähler moduli are frozen.

Let us provide independent support for this claim from heterotic-Type I duality[6]. It is known that a D9-brane wrapping the Calabi-Yau $\mathcal{M}$ with Kähler form J^I and a $U(1)$ bundle with field strength F_m is supersymmetric if it satisfies the MMMS condition [49]

$$\begin{aligned}\frac{1}{2}\int_{\mathcal{M}} J^I\wedge J^I\wedge \mathcal{F}_m - \frac{1}{3!}\int_{\mathcal{M}} \mathcal{F}_m\wedge\mathcal{F}_m\wedge\mathcal{F}_m = \\ -\tan\theta\left(\frac{1}{2}\int_{\mathcal{M}} \mathcal{F}_m\wedge\mathcal{F}_m\wedge J^I - \frac{1}{3!}\int_{\mathcal{M}} J^I\wedge J^I\wedge J^I\right)\end{aligned} \tag{73}$$

with $\mathcal{F}_m = 2\pi\alpha'\, F_m$. For $\theta = 0$ this looks quite similar to (72), except that the MMMS condition is at string tree level and the $\mathcal{F}_m^3$ term is an α' correction. Applying the heterotic-Type I string duality relations [50]

$$\begin{aligned} e^{\phi^I_{10}} &= e^{-\phi^H_{10}}, \\ J^I &= J^H\, e^{-\phi^H_{10}}, \end{aligned} \tag{74}$$

the MMMS condition leads to

$$\frac{e^{-2\,\phi^H_{10}}}{2}\int_{\mathcal{M}} J^H\wedge J^H\wedge F_m - \frac{(2\pi\alpha')^2}{3!}\int_{\mathcal{M}} F_m\wedge F_m\wedge F_m = 0. \tag{75}$$

Qualitatively, this has precisely the form of the abelian part of (72) supporting our claim for the one-loop correction to the DUY equation. A derivation of this equation via an FI term in an effective four-dimensional theory has been carried out in [51].

To our knowledge the inclusion of non-abelian gauge fields and of the curvature term into the MMMS equation (73) is not known (see for instance [52, 53] for some proposals). The discussion above implies that this generalisation is likely to involve the trace parts in (72). In this respect it would be interesting to also compute the FI-terms for the $SO(32)$ heterotic string [54].

Which further corrections to the DUY condition do we have to expect? From the supergravity analysis of the D-term, it is clear that there cannot be any higher string-loop contributions, well in accord with the fact that the MMMS-condition in Type I is exact in α'-perturbation theory. Moreover, it is known [55] that there are no one-loop Fayet-Iliopoulos terms on the Type I side. Consequently, S-duality dictates that the DUY equation is also exact in sigma-model perturbation theory. However, there might be additional non-pertrubative corrections which are beyond the scope of this paper.

[6]Though this argument in the strong sense is valid for the $SO(32)$ theories, qualitatively it should also teach us something about the $E_8\times E_8$ heterotic string.

Note that the corrected DUY equation has the interesting prospect that the values of the frozen Kähler moduli depend on the value of the string coupling constant and vice versa. Of course whether such minima exist depends on the various sign factors in (72). In particular for certain fluxes it allows one to freeze combinations of Kähler moduli and the dilaton such that both are still in the perturbative regime. Moreover, it is also possible to have $U(1)$ bundles on non-degenerate Calabi-Yau manifolds with $h_{11}=1$. In the next section we will provide an example which precisely shows all these features.

3 Bundles with structure group $SU(4)\times U(1)$

3.1 $SU(4)$ bundles

In the remainder of this article, we will apply the results described so far to the construction of explicit models. As a warm-up, in this section we consider the $E_8\times E_8$ heterotic string compactified on a Calabi-Yau manifold $\mathcal{M}$ equipped with the specific class of bundles

$$W = V \oplus L \tag{76}$$

with structure group $G=SU(4)\times U(1)$. Let us first provide the general expressions for the massless spectrum in four dimensions.

Embedding this structure group into one of the E_8 factors leads to the breaking to $H=SU(5)\times U(1)_X$, where the adjoint of E_8 decomposes as follows into $G\times H$ representations (note that this notation is a little too sloppy since the type (i) $U(1)$ factors in G and H are identical)

$$\mathbf{248} \overset{SU(4)\times SU(5)\times U(1)_X}{\longrightarrow} \left\{ \begin{array}{c} (\mathbf{15},\mathbf{1})_0 \\ (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{10})_4 + (\mathbf{1},\overline{\mathbf{10}})_{-4} + (\mathbf{1},\mathbf{24})_0 \\ (\mathbf{4},\mathbf{1})_{-5} + (\mathbf{4},\overline{\mathbf{5}})_3 + (\mathbf{4},\mathbf{10})_{-1} \\ (\overline{\mathbf{4}},\mathbf{1})_5 + (\overline{\mathbf{4}},\mathbf{5})_{-3} + (\overline{\mathbf{4}},\overline{\mathbf{10}})_1 \\ (\mathbf{6},\mathbf{5})_2 + (\mathbf{6},\overline{\mathbf{5}})_{-2} \end{array} \right\}. \tag{77}$$

As shown in Table 1, from (77) one can immediately read off by which cohomology classes the massless spectrum is determined.
From this embedding of the structure group, we can determine the resulting tadpole cancellation condition by computing the traces

$$\begin{aligned} \mathrm{tr}(\overline{F}^2) &= \frac{1}{30}\,\mathrm{Tr}(\overline{F}^2) = \frac{1}{30}\sum_{\mathrm{repr}\,i}(\mathrm{tr}_i \overline{F}^2_{SU(4)} + \mathrm{tr}_i \overline{F}^2_{U(1)}) \\ &= 2\,\mathrm{tr}_f^{SU(4)}(\overline{F}^2_{SU(4)}) + 40\,\overline{F}^2_{U(1)} = 4\,(2\pi)^2\,(-c_2(V) + 10\,c_1^2(L)), \quad (78) \\ \mathrm{tr}(\overline{R}^2) &= 2\,\mathrm{tr}_f^{SU(3)}(\overline{R}^2) = -4\,(2\pi)^2\,c_2(T), \end{aligned}$$

reps.	Cohomology
$\mathbf{10}_{-1}$	$H^*(\mathcal{M}, V\otimes L^{-1})$
$\mathbf{10}_{4}$	$H^*(\mathcal{M}, L^{4})$
$\overline{\mathbf{5}}_{3}$	$H^*(\mathcal{M}, V\otimes L^{3})$
$\overline{\mathbf{5}}_{-2}$	$H^*(\mathcal{M}, \bigwedge^2 V\otimes L^{-2})$
$\mathbf{1}_{-5}$	$H^*(\mathcal{M}, V\otimes L^{-5})$

Table 1: Massless spectrum of $H = SU(5)\times U(1)_X$ models.

yielding

$$c_2(V) - 10\, c_1^2(L) = c_2(T). \tag{79}$$

Let us stress that here indeed the coefficient in front of $c_1^2(L)$ is not equal to one-half, as one might have expected. One might worry that this is simply a consequence of the normalisation of $U(1)_X$ in the sense that with the usual normalisation $\mathrm{tr}_{E_8}(Q^2)=2$ the coefficient would indeed be one. But this is not the case, as we cannot change the powers of the line bundle L in Table 1 accordingly. These powers have to be integers. We will see in the next section that this result is consistent with the abelian anomalies, which, due to the general results presented in section 2.3, must also contain a certain combination of the traces in (78).

The net-number of chiral multiplets is given by the Euler characteristic of the respective bundle, eq. (13). Note that extra gauge bosons are counted by $H^*(\mathcal{M},\mathcal{O})$, which can only appear in Table 1 if L^4 is the trivial bundle $\mathcal{O}$, i.e. $c_1(L)=0$. Clearly in this case the gauge symmetry is extended to $SO(10)$, which is precisely the commutant of $SU(4)$ in E_8. We will see that for the case that more $U(1)$ bundles are involved the patterns of gauge symmetry enhancement are more intricate.

It is now a straightforward exercise to compute the four-dimensional gauge anomalies.

• The non-abelian $SU(5)^3$ anomaly is proportional to

$$A_{SU(5)^3} = \chi(\mathcal{M}, V\otimes L^{-1}) + \chi(\mathcal{M}, L^4) - \chi(\mathcal{M}, V\otimes L^{3}) - \chi(\mathcal{M}, \textstyle\bigwedge^2 V\otimes L^{-2}). \tag{80}$$

As expected just using (13) and (19), this anomaly vanishes (even without invoking the tadpole cancellation condition).

• The mixed abelian-gravitational anomaly $U(1)_X - G^2_{\mu\nu}$ however does not directly vanish and is given by

$$A_{U(1)-G^2_{\mu\nu}} \;=\; -10\,\chi(\mathcal{M}, V\otimes L^{-1}) + 40\,\chi(\mathcal{M}, L^4) + 15\,\chi(\mathcal{M}, V\otimes L^{3}) -$$

$$
\begin{aligned}
& 10\,\chi(\mathcal{M},\textstyle\bigwedge^2 V\otimes L^{-2}) - 5\,\chi(\mathcal{M},V\otimes L^{-5}) \\
= \;& 10\int_{\mathcal{M}} c_1(L)\,\left[12(-c_2(V)+10\,c_1^2(L)) + 5\,c_2(T)\right]. \qquad (81)
\end{aligned}
$$

• Similarly the mixed abelian-non-abelian anomaly $U(1)_X - SU(5)^2$ takes the form

$$
\begin{aligned}
A_{U(1)-SU(5)^2} &= -3\,\chi(\mathcal{M},V\otimes L^{-1}) + 12\,\chi(\mathcal{M},L^4) + 3\,\chi(\mathcal{M},V\otimes L^3) - \\
&\quad 2\,\chi(\mathcal{M},\textstyle\bigwedge^2 V\otimes L^{-2}) \\
&= 10\int_{\mathcal{M}} c_1(L)\,\left[2(-\,c_2(V)+10\,c_1^2(L)) + c_2(T)\right]. \qquad (82)
\end{aligned}
$$

• Finally for the $U(1)_X^3$ anomaly one obtains

$$
\begin{aligned}
A_{U(1)^3} &= -10\,\chi(\mathcal{M},V\otimes L^{-1}) + 640\,\chi(\mathcal{M},L^4) + 135\,\chi(\mathcal{M},V\otimes L^3) - \\
&\quad 40\,\chi(\mathcal{M},\textstyle\bigwedge^2 V\otimes L^{-2}) - 125\,\chi(\mathcal{M},V\otimes L^{-5}) \\
&= 200\int_{\mathcal{M}} c_1(L)\,\left[6(-c_2(V)+10\,c_1^2(L)) + 40c_1^2(L) + 3\,c_2(T)\right]. \qquad (83)
\end{aligned}
$$

These results are in complete agreement with the general expressions (27 - 29) reviewed in section 2.3 if one uses (78) to rewrite them in terms of traces. Note that the integrands only vanish if $c_1(L) = 0$, in which case the gauge group is enhanced to $SO(10)$.

3.2 Example: An $SU(4)$ bundle on the Quintic

As one of its virtues, the 1-loop correction of the DUY equation provides us with supersymmetric models inside the Kähler cone even for internal manifolds with $h_{11} = 1$ due to the fixing of only a linear combination of the Kähler moduli and the dilaton. Let us illustrate this by constructing a simple though not realistic model on the mother of all Calabi-Yau compactifications, the Quintic, with Hodge numbers $(h_{21}, h_{11}) = (101, 1)$, intersection form

$$I_3 = 5\,\eta^3 \qquad (84)$$

and

$$c_2(T) = 10\,\eta^2. \qquad (85)$$

As an example of the above construction, consider a bundle of the form

$$W = V_1 \oplus V_2 \oplus L, \qquad (86)$$

where the $SU(4)$ bundle V_1 and the line bundle L are embedded into the visible E_8 factor and the $SU(4)$ bundle V_2 into the second E_8 bundle. Concretely, we define both vector bundles as the cohomology of the monad

$$0 \to \mathcal{O}|_{\mathcal{M}} \to \mathcal{O}(1)^{\oplus 5} \oplus O(3)|_{\mathcal{M}} \to \mathcal{O}(8)|_{\mathcal{M}} \to 0 \qquad (87)$$

and pick $L = \mathcal{O}(2)$. It is easy to check that this choice of data satisfies the tadpole equation

$$c_2(V_1) + c_2(V_2) - 10\, c_1^2(L) = c_2(T). \tag{88}$$

In particular, this implies that

$$\frac{1}{(2\pi)^2}(\mathrm{tr}\overline{F_1}^2 - \frac{1}{2}\mathrm{tr}\overline{R}^2) = -2c_2(T) + 4c_2(V_2). \tag{89}$$

Upon defining the Kähler form $J = \ell_s^2\, r\, \eta$, we thus arrive at the DUY equation

$$r^2 = 10\, e^{2\phi_{10}}, \tag{90}$$

one solution of which indeed corresponds to the radius of the Quintic stabilized inside the Kähler cone at a value dictated by the concrete string coupling. It also admits solutions where both the string coupling and the radius are in the weakly coupled regime. Choosing for instance $g_s = 0.8$ yields $r = 2.52$. Let us stress once again that the naive tree level DUY equation $\int J \wedge J \wedge c_1(L) = 0$ would have stabilized the radius at $r = 0$.

Note also that the result (90) depends crucially on the presence of the hidden bundle V_2. This model is supposed to serve just as an illustration of the new model building possibilities and that more systematic searches may turn out to be fruitful. Suffice it here to merely add for completeness the not quite realistic chiral $SU(5) \times U(1)_X$ spectrum in Table 2, where the anomalous $U(1)_X$ only survives as a global symmetry.

reps.	χ
$\mathbf{10}_{-1}$	$\chi(\mathcal{M}, V \otimes L^{-1}) = 290$
$\mathbf{10}_{4}$	$\chi(\mathcal{M}, L^{4}) = 460$
$\overline{\mathbf{5}}_{3}$	$\chi(\mathcal{M}, V \otimes L^{3}) = 170$
$\overline{\mathbf{5}}_{-2}$	$\chi(\mathcal{M}, \bigwedge^2 V \otimes L^{-2}) = 580$
$\mathbf{1}_{-5}$	$\chi(\mathcal{M}, V \otimes L^{-5}) = -2150$

Table 2: Chiral spectrum of an $H = SU(5) \times U(1)_X$ model.

3.3 $U(4)$ bundles

Instead of starting with an $SU(4)$ bundle, we could have also used a $U(4)$ bundle. Such a construction has been considered in [17] before. One starts with a bundle

$$W = V \oplus L^{-1}, \quad \text{with } c_1(V) = c_1(L), \;\; \mathrm{rank}(V) = 4, \tag{91}$$

which has structure group $SU(4)\times U(1)$.[7] This bundle W can now be embedded into an $SU(5)$ subgroup of E_8 so that the commutant is again $SU(5)\times U(1)$. We embed the $U(1)$ bundle such that

$$Q_1 = (1,1,1,1,-4), \tag{92}$$

implying that the matrix $\mathcal{Q}$ defined in (26) is simply

$$\mathcal{Q} = Q_1(V) + Q_1(L) = 5. \tag{93}$$

In fact, consider the breaking of the original structure group $SU(5)\to U(4)\times U(1)$ and the corresponding decomposition of $(\mathbf{5},\mathbf{10})\to(\mathbf{4},\mathbf{10})_{-1}+(\mathbf{1},\mathbf{10})_4$ to read off the unique charge assignments of V and L. Consequently, the massless spectrum is now given by the cohomology classes listed in Table 3.

reps.	Cohom.
$\mathbf{10}_{-1}$	$H^*(\mathcal{M},V)$
$\mathbf{10}_{4}$	$H^*(\mathcal{M},L^{-1})$
$\overline{\mathbf{5}}_{3}$	$H^*(\mathcal{M},V\otimes L^{-1})$
$\overline{\mathbf{5}}_{-2}$	$H^*(\mathcal{M},\bigwedge^2 V)$
$\mathbf{1}_{-5}$	$H^*(\mathcal{M},V\otimes L)$

Table 3: Massless spectrum of $H=SU(5)\times U(1)_X$ models.

The resulting tadpole cancellation condition reads

$$c_2(V) - c_1^2(V) = c_2(T). \tag{94}$$

Similarly to the former case, one can show that all non-abelian gauge anomalies cancel and that the abelian ones,

$$\begin{aligned}
A_{U(1)-G^2_{\mu\nu}} &= -\frac{5}{2}\int_{\mathcal{M}} c_1(L)\,\Big[12\Big(-c_2(V)+\,c_1^2(L)\Big)+5\,c_2(T)\Big]\,,\\
A_{U(1)-SU(5)^2} &= -\frac{5}{2}\int_{\mathcal{M}} c_1(L)\,\Big[2\Big(-c_2(V)+\,c_1^2(L)\Big)+c_2(T)\Big]\,,\\
A_{U(1)^3} &= -25\int_{\mathcal{M}} c_1(L)\,\Big[12\Big(-c_2(V)+\,c_1^2(L)\Big)+5c_1^2(L)+6\,c_2(T)\Big]\,,
\end{aligned} \tag{95}$$

are cancelled by a Green-Schwarz mechanism [8]. Taking into account $\mathrm{tr}_{E_8}(Q_1^2)=40$, these anomalies are consistent with the general result (54,55,56).

[7]Note that the two abelian factors in V and L are correlated via (91).

[8]Our result for the abelian anomalies disagrees with the values given in [18].

3.4 Example: A $U(4)$ bundle on a CICY

As a concrete example we present a $U(4)$ bundle on the Calabi-Yau three-fold

$$\mathcal{M} = \begin{matrix} \mathbb{P}_3 \\ \mathbb{P}_1 \end{matrix} \left[\begin{matrix} 4 \\ 2 \end{matrix} \right] \tag{96}$$

with Hodge numbers $(h_{21}, h_{11}) = (86, 2)$. Let η_1 denote the two-form on $\mathbb{P}_3$ and η_2 the two-form on $\mathbb{P}_1$. The resulting Stanley-Reisner ideal $SR = \{\eta_1^4, \eta_2^2\}$ on the ambient space eventually determines the intersection form on the Calabi-Yau

$$I_3 = 2\eta_1^3 + 4\,\eta_1^2\eta_2. \tag{97}$$

Due to $\eta_2^2 = 0$ there are two four-forms on $\mathcal{M}$, namely $\{\eta_1^2, \eta_1\eta_2\}$. The total Chern class of the manifold is given by

$$c(T) = \frac{(1+\eta_1)^4\,(1+\eta_2)^2}{(1+4\eta_1+2\eta_2)}, \tag{98}$$

leading in particular to

$$c_2(T) = 6\eta_1^2 + 8\eta_1\eta_2. \tag{99}$$

To see how the requirement of supersymmetry relates the expectation value of the dilaton and the Kähler moduli of the Calabi-Yau, we use the tadpole cancellation condition to arrive at $\frac{1}{(2\pi)^2}(\mathrm{tr}(\overline{F})^2 - \frac{1}{2}\mathrm{tr}(\overline{R})^2) = -2\,c_2(T)$ for this kind of construction. For a general line bundle $L = \mathcal{O}(m,n)$ and a general Kähler class $J = \ell_s^2\,(r_1\eta_1 + r_2\eta_2)$ with $r_{1,2} \geq 0$ the 1-loop corrected DUY condition reads in this case

$$r_1\,\left[(2n+m)r_1 + 4m\,r_2\right] + \frac{1}{2}(11m+6n)\,e^{2\phi_{10}} = 0, \tag{100}$$

which leaves enough room to stabilize the ratio of the Kähler moduli inside the Kähler cone for any given value of the string coupling constant by a suitable choice of the line bundle. The volume of the Calabi-Yau manifold is given by

$$\mathrm{Vol}(\mathcal{M}) = \frac{1}{6}\int_{\mathcal{M}} J^3 = \ell_s^6\,(2\,r_1^3 + 12\,r_1^2 r_2). \tag{101}$$

Now we take $L = \mathcal{O}(-2,2)$ and the $U(4)$ bundle V defined via the exact sequence

$$0 \to V \to \mathcal{O}(1,0)^{\oplus 2} \oplus \mathcal{O}(0,1)^{\oplus 2} \oplus \mathcal{O}(1,1)^{\oplus 2}|_{\mathcal{M}} \xrightarrow{f} \mathcal{O}(4,1) \oplus \mathcal{O}(2,1)|_{\mathcal{M}} \to 0. \tag{102}$$

One can choose the map f such that it does not degenerate at any point on $\mathcal{M}$, which ensures that the exact sequence (102) really defines a bona-fide vector

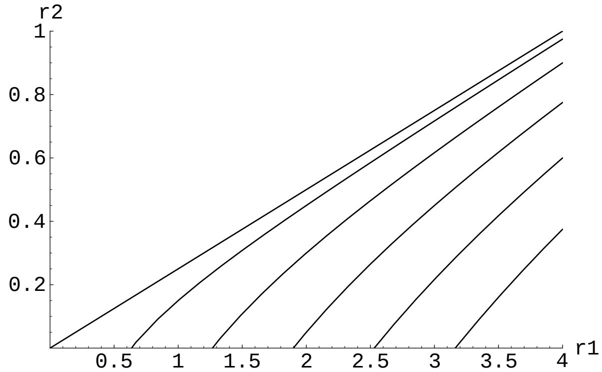

Figure 2: The plot shows the Kähler moduli (r_1, r_2) for the values $4g_s \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$ of the string coupling constant.

bundle. As with all these constructions we do not know how to proof that the bundle is really stable.

One can easily check that

$$\mathrm{ch}_2(V) + \frac{1}{2}c_1^2(L^{-1}) = -c_2(T) \tag{103}$$

is satisfied. Moreover, the DUY condition imposes the constraint

$$r_1^2 - 4\, r_1\, r_2 = \frac{5}{2}\, e^{2\phi_{10}} \tag{104}$$

on the two radii and the dilaton. The solutions to this constraint for certain values of the string coupling constant are shown in Figure 2.

The next step is to compute the massless spectrum. For the chiral one it is sufficient to compute just the various Euler characteristics, whereas for determining also the complete non-chiral massless spectrum, one has to compute the various cohomology classes by tracing through the long exact sequences in cohomology [32]. The essential input here is that the cohomology classes of line bundles $\mathcal{O}(m,n)$ over the ambient space $\mathbb{P}_3 \times \mathbb{P}_1$ can be determined from Bott's formula

$$\begin{aligned}
h^0 &= \binom{m+3}{3}(n+1) \qquad && \text{for } m,n \geq 0, \\
h^1 &= \binom{m+3}{3}(-n-1) \qquad && \text{for } m \geq 0, n \leq -2, \\
h^2 &= 0 \qquad && \text{for all } m,n, \\
h^3 &= \binom{-m-1}{3}(n+1) \qquad && \text{for } m \leq -4, n \geq 0,
\end{aligned} \tag{105}$$

$$h^4 \;=\; \binom{-m-1}{3}(-n-1) \qquad \text{for } m \le -4, n \le -2.$$

The results for the massless spectrum are listed in Table 4.

reps.	Cohom.	χ
$\mathbf{10}_{-1}$	$H^*(\mathcal{M}, V) = (0, 62, 0, 0)$	-62
$\mathbf{10}_{4}$	$H^*(\mathcal{M}, L^{-1}) = (0, 10, 0, 0)$	-10
$\overline{\mathbf{5}}_{3}$	$H^*(\mathcal{M}, V \otimes L^{-1}) = (0, 62, 0, 0)$	-62
$\overline{\mathbf{5}}_{-2}$	$H^*(\mathcal{M}, \bigwedge^2 V) = (x, 10 + x + y, y, 0)$	-10
$\mathbf{1}_{-5}$	$H^*(\mathcal{M}, V \otimes L) = (0, 44, 62, 0)$	18

Table 4: Massless spectrum of an $SU(5) \times U(1)$ model. For $H^*(\mathcal{M}, \bigwedge^2 V)$ the sequence computation is highly non-trivial with a vast number of explicit maps to be investigated, and we do not perform the explicit calculation here.

Since L is not trivial, the $U(1)_X$ symmetry becomes massive via the Green-Schwarz mechanism but survives as a perturbative global symmetry. Taking also this global quantum number into account we have obtained an $SU(5)$ GUT model with 62 generations and some exotic chiral states. Ignoring the $U(1)_X$ charge, we would say that we have a model with 72 generations. Note that $\chi(\mathcal{M}, V) = \chi(\mathcal{M}, V \otimes L^{-1})$ is just an accident for this concrete model, but shows that extra relations for the massless spectrum can be obtained in this class of models. Of course such extra conditions have to be imposed in general to guarantee the absence of additional exotic chiral matter.

4 Bundles with structure group $SU(4) \times U(1)^2$

By embedding a second $U(1)$ bundle into the observable $SU(5)$, one can break the $SU(5)$ to the Standard Model gauge symmetry. Therefore, we now consider an $SU(4) \times U(1) \times U(1)$ bundle

$$W = V \oplus L_1 \oplus L_2 \tag{106}$$

and a $U(4) \times U(1) \times U(1)$ bundle

$$W = V \oplus L_1^{-1} \oplus L_2^{-1} \tag{107}$$

with $c_1(W) = 0$, respectively. In this latter case, the embedding of the two $U(1)$ bundles into $SU(6)$ is given by

$$Q_1 = (-1, -1, -1, -1, 4, 0), \quad Q_2 = (1, 1, 1, 1, 1, -5), \tag{108}$$

with $\mathrm{tr}_{E_8}(Q_1^2)=40$ and $\mathrm{tr}_{E_8}(Q_2^2)=60$. This leads to

$$\mathcal{Q}=\begin{pmatrix} -5 & -1 \\ 0 & 6 \end{pmatrix}. \tag{109}$$

4.1 The massless spectrum and gauge enhancement

The commutant in this case is $H = SU(3)\times SU(2)\times U(1)_X\times U(1)_{Y'}$ and the resulting decomposition of the adjoint representation of E_8 reads

$$\mathbf{248} \xrightarrow{SU(4)\times SU(3)\times SU(2)\times U(1)^2} \left\{ \begin{array}{c} (\mathbf{15},\mathbf{1},\mathbf{1})_{0,0} \\ 2\times(\mathbf{1},\mathbf{1},\mathbf{1})_{0,0}+(\mathbf{1},\mathbf{8},\mathbf{1})_{0,0}+(\mathbf{1},\mathbf{1},\mathbf{3})_{0,0} \\ (\mathbf{1},\mathbf{3},\mathbf{2})_{0,-5}+c.c. \\ (\mathbf{1},\mathbf{3},\mathbf{2})_{4,1}+(\mathbf{1},\overline{\mathbf{3}},\mathbf{1})_{4,-4}+(\mathbf{1},\mathbf{1},\mathbf{1})_{4,6}+c.c. \\ (\mathbf{4},\mathbf{3},\mathbf{2})_{-1,1}+(\mathbf{4},\overline{\mathbf{3}},\mathbf{1})_{-1,-4}+(\mathbf{4},\mathbf{1},\mathbf{1})_{-1,6}+c.c. \\ (\mathbf{4},\overline{\mathbf{3}},\mathbf{1})_{3,2}+(\mathbf{4},\mathbf{1},\mathbf{2})_{3,-3}+(\mathbf{4},\mathbf{1},\mathbf{1})_{-5,0}+c.c \\ (\mathbf{6},\overline{\mathbf{3}},\mathbf{1})_{-2,2}+(\mathbf{6},\mathbf{1},\mathbf{2})_{-2,-3}+c.c. \end{array} \right\} \tag{110}$$

Note that the $U(1)$ charges are proportional to the $U(1)_X$ and $U(1)_{Y'}$ charges in the flipped $SU(5)$ GUT model. The (possibly anomalous) hypercharge $U(1)_Y$ and the $U(1)_{B-L}$ charge are given by the linear combinations

$$Q_Y=-\frac{1}{15}\,Q_{Y'}-\frac{2}{5}\,Q_X, \qquad Q_{B-L}=\frac{2}{15}\,Q_{Y'}-\frac{1}{5}\,Q_X. \tag{111}$$

The massless spectrum is counted by the cohomology classes in Table 5. The resulting tadpole cancellation condition reads

$$c_2(V)-10\,c_1^2(L_1)-15\,c_1^2(L_2)=c_2(T) \tag{112}$$

for the $SU(4)\times U(1)^2$ bundle and

$$-\,\mathrm{ch}_2(V)-\frac{1}{2}\sum_{i=1}^{2}c_1^2(L_i)=c_2(T) \tag{113}$$

for the $U(4)\times U(1)^2$ bundle. For generic first Chern classes $c_1(L_1)$ and $c_1(L_2)$, the two $U(1)$ gauge symmetries are anomalous and gain a mass via the Green-Schwarz mechanism. Therefore, the generic unbroken gauge symmetry is $SU(3)\times SU(2)$. By computing the various anomalies, one can show that the linear combination

$$U(1)_f \simeq \kappa_1\,U(1)_X+\kappa_2\,U(1)_{Y'} \tag{114}$$

is anomaly-free precisely if the first Chern classes of the two line bundles for the $SU(4)\times U(1)^2$ case satisfy the relation

$$2\kappa_1\,c_1(L_1)+3\kappa_2\,c_1(L_2)=0 \tag{115}$$

reps.	$SU(4)\times U(1)^2$	$U(4)\times U(1)^2$
$(\mathbf{3},\mathbf{2})_{-1,1}$	$H^*(\mathcal{M}, V\otimes L_1^{-1}\otimes L_2)$	$H^*(\mathcal{M}, V)$
$(\overline{\mathbf{3}},\mathbf{1})_{-1,-4}$	$H^*(\mathcal{M}, V\otimes L_1^{-1}\otimes L_2^{-4})$	$H^*(\mathcal{M}, V\otimes L_2^{-1})$
$(\mathbf{1},\mathbf{1})_{-1,6}$	$H^*(\mathcal{M}, V\otimes L_1^{-1}\otimes L_2^{6})$	$H^*(\mathcal{M}, V\otimes L_2)$
$(\overline{\mathbf{3}},\mathbf{1})_{3,2}$	$H^*(\mathcal{M}, V\otimes L_1^{3}\otimes L_2^{2})$	$H^*(\mathcal{M}, V\otimes L_1^{-1})$
$(\mathbf{1},\mathbf{2})_{3,-3}$	$H^*(\mathcal{M}, V\otimes L_1^{3}\otimes L_2^{-3})$	$H^*(\mathcal{M}, V\otimes L_1^{-1}\otimes L_2^{-1})$
$(\mathbf{1},\mathbf{1})_{-5,0}$	$H^*(\mathcal{M}, V\otimes L_1^{-5})$	$H^*(\mathcal{M}, V\otimes L_1)$
$(\overline{\mathbf{3}},\mathbf{1})_{-2,2}$	$H^*(\mathcal{M}, \bigwedge^2 V\otimes L_1^{-2}\otimes L_2^{2})$	$H^*(\mathcal{M}, \bigwedge^2 V)$
$(\mathbf{1},\mathbf{2})_{-2,-3}$	$H^*(\mathcal{M}, \bigwedge^2 V\otimes L_1^{-2}\otimes L_2^{-3})$	$H^*(\mathcal{M}, \bigwedge^2 V\otimes L_2^{-1})$
$(\mathbf{3},\mathbf{2})_{4,1}$	$H^*(\mathcal{M}, L_1^{4}\otimes L_2)$	$H^*(\mathcal{M}, L_1^{-1})$
$(\overline{\mathbf{3}},\mathbf{1})_{4,-4}$	$H^*(\mathcal{M}, L_1^{4}\otimes L_2^{-4})$	$H^*(\mathcal{M}, L_1^{-1}\otimes L_2^{-1})$
$(\mathbf{1},\mathbf{1})_{4,6}$	$H^*(\mathcal{M}, L_1^{4}\otimes L_2^{6})$	$H^*(\mathcal{M}, L_1^{-1}\otimes L_2)$
$(\mathbf{3},\mathbf{2})_{0,-5}$	$H^*(\mathcal{M}, L_2^{-5})$	$H^*(\mathcal{M}, L_2^{-1})$

Table 5: Massless spectrum of $H = SU(3)\times SU(2)\times U(1)_X\times U(1)_{Y'}$ models.

and for the $U(4)\times U(1)^2$ case

$$5\kappa_1\, c_1(L_1) - (6\kappa_2-\kappa_1)\, c_1(L_2) = 0. \tag{116}$$

In the $SU(4)\times U(1)^2$ case, for certain values of the parameters κ_1, κ_2 some of the line bundles $L_1^4\otimes L_2$, $L_1^4\otimes L_2^{-4}$, $L_1^4\otimes L_2^6$ and L_2^{-5} appearing in Table 5 become trivial and signal a non-abelian enhancement of the gauge symmetry. For the $U(4)\times U(1)^2$ bundles the situation is of course completely similar. The four, respectively five for all line bundles trivial, possible non-abelian enhancements of $SU(3)\times SU(2)$ are depicted in Figure 1. This shows that not only the expected $SO(10)$ and $SU(5)$ gauge groups are possible, but also other gauge groups containing $SU(3)\times SU(2)\times U(1)^2$ as a subgroup.

Another way of understanding these gauge symmetry enhancements is by observing that the linear relations (115,116) for the two line bundles imply that the structure group is reduced to $SU(4)\times U(1)$, which of course enhances the commutant. For a generic linear relation (115,116) the extra $U(1)$ appearing in the commutatant is of type (ii). What the commutant precisely is, depends on how the $U(1)$ is embedded into $SO(10)$, but such a group theoretic analysis is not necessary as one can read off the enhanced gauge symmetries simply from Table 5.

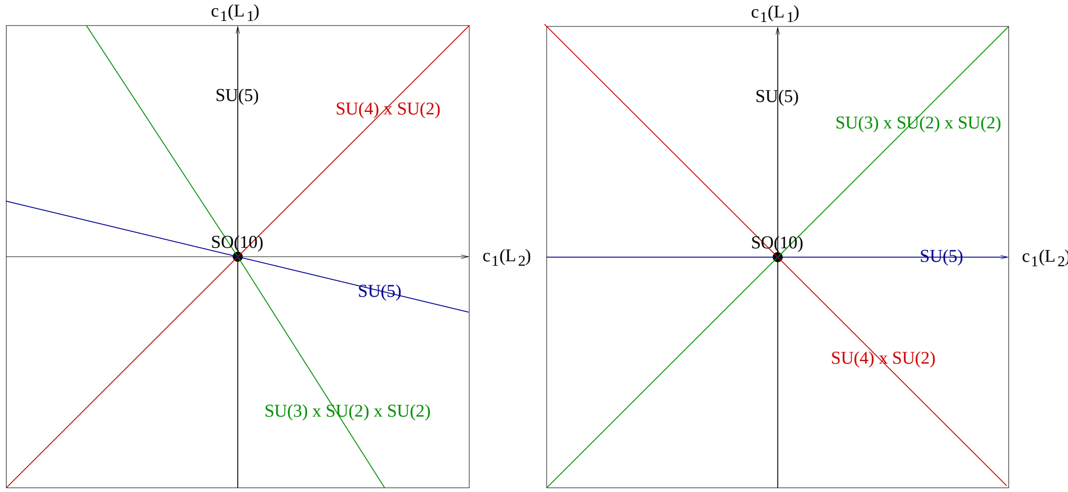

Figure 3: Gauge symmetry enhancement for bundles with structure group $SU(4) \times U(1)^2$. On generic lines through the origin the gauge symmetry is enhanced to $SU(3) \times SU(2) \times U(1)$ while for the specific values shown one gets even non-abelian enhancement. The left image shows the loci of non-abelian enhancement in the $(c_1(L_2), c_1(L_1))$-plane for $SU(4) \times U(1)^2$ and the right image for $U(4) \times U(1)^2$.

For instance, if respectively $L_1^4 \otimes L_2^{-4}$ or $L_1^{-1} \otimes L_2^{-1}$ become trivial, the $U(1)_{B-L}$ is anomaly-free and enhances the $SU(3)$ gauge symmetry to $SU(4)$. Let us show a concrete example of such a bundle.

4.2 Example: A bundle with $SU(4) \times SU(2)$ gauge symmetry

We consider the same CICY as in section 3.4 and choose the two line bundles as

$$L_2 = L_1^{-1} = \mathcal{O}(-2, 2). \tag{117}$$

Therefore, the Kähler moduli and the dilaton are again related by the constraint $r_1^2 - 4 r_1 r_2 = \frac{5}{2} e^{2\phi_{10}}$. From our general analysis the observable gauge symmetry is $G = SU(4) \times SU(2)$ and one linear combination of the two $U(1)$s is anomalous and gets a mass via the Green-Schwarz mechanism. In this case the $U(4)$ bundle is in fact an $SU(4)$ bundle and one can satisfy the tadpole cancellation condition by choosing

$$0 \to \ V \to \mathcal{O}(1,0)^{\oplus 5} \xrightarrow{f} \mathcal{O}(5,0)|_{\mathcal{M}} \to 0. \tag{118}$$

This bundle has the property $H^0(\mathcal{M}, V) = H^3(\mathcal{M}, V) = 0$, which is a necessary

condition for the bundle to be stable. The resulting chiral massless spectrum is listed in Table 6.

reps.	χ
$(\mathbf{4},\mathbf{2})$	$\chi(\mathcal{M},V)=-40$
$(\overline{\mathbf{4}},\mathbf{1})$	$\chi(\mathcal{M},V\otimes L_2^{-1})=-110$
$(\overline{\mathbf{4}},\mathbf{1})$	$\chi(\mathcal{M},V\otimes L_2)=30$
$(\mathbf{6},\mathbf{2})$	$\chi(\mathcal{M},L_2^{-1})=-10$
$(\mathbf{1},\mathbf{2})$	$\chi(\mathcal{M},\bigwedge^2 V\otimes L_2^{-1})=-140$

Table 6: Massless spectrum of an $SU(4)\times SU(2)$ model.

Clearly, this is not a realistic model, but it shows that non-trivial models with extra enhanced gauge symmetry can be obtained in this set-up.

5 Bundles with structure group $SU(3)\times U(1)^3$

5.1 The massless spectrum and gauge enhancement

Let us explore further the model building possibilities several line bundles bring about and consider the embedding of a bundle of the type

$$W=V\oplus L_1\oplus L_2\oplus L_3 \tag{119}$$

with structure group $G=SU(3)\times U(1)\times U(1)\times U(1)$. We thus break E_8 down to $H=SU(3)\times SU(2)\times U(1)_Z\times U(1)_X\times U(1)_{Y'}$ by replacing the internal $SU(4)$ bundle of the previous example by an $SU(3)\times U(1)_Z$ bundle. Alternatively, one can again choose the bundle W to be of the form

$$W=V\oplus L_1^{-1}\oplus L_2^{-1}\oplus L_3^{-1} \tag{120}$$

and the structure group of V to be $U(3)$. In this latter case, the embedding of the three $U(1)$ bundles into $SU(6)$ is given by

$$Q_1=(1,1,1,-3,0,0),\ Q_2=(-1,-1,-1,-1,4,0),\ Q_3=(1,1,1,1,1,-5) \tag{121}$$

with $\mathrm{tr}_{E_8}(Q_1^2)=24$, $\mathrm{tr}_{E_8}(Q_2^2)=40$ and $\mathrm{tr}_{E_8}(Q_3^2)=60$. This leads to

$$\mathcal{Q}=\begin{pmatrix}4&1&1\\0&-5&-1\\0&0&6\end{pmatrix}. \tag{122}$$

The massless spectrum for both cases is counted by the respective cohomology classes in Table 10 of Appendix B. The resulting tadpole cancellation condition reads

$$c_2(V) - 6\, c_1^2(L_1) - 10\, c_1^2(L_2) - 15\, c_1^2(L_3) = c_2(T) \tag{123}$$

for the $SU(3) \times U(1)^3$ bundle and

$$-\operatorname{ch}_2(V) - \frac{1}{2}\sum_{i=1}^{3} c_1^2(L_i) = c_2(T) \tag{124}$$

for the $U(3) \times U(1)^3$ bundle.

For generic first Chern classes $c_1(L_1)$, $c_1(L_2)$ and $c_1(L_3)$ the three $U(1)$ gauge symmetries are anomalous and gain a mass via the Green-Schwarz mechanism. Therefore, the generic gauge symmetry is $SU(3)\times SU(2)$. However, for particular choices of the bundle data we encounter a rich pattern of gauge enhancement, as we will now discuss systematically.

The computation of the various anomalies for the $SU(3) \times U(1)^3$ case reveals that the linear combination

$$U(1)_f = \kappa_1\, U(1)_Z + \kappa_2\, U(1)_X + \kappa_3\, U(1)_{Y'} \tag{125}$$

is anomaly-free precisely if the first Chern classes of the line bundles satisfy

$$6\kappa_1\, c_1(L_1) + 10\kappa_2\, c_1(L_2) + 15\kappa_3\, c_1(L_3) = 0. \tag{126}$$

The corresponding constraint for the $U(3) \times U(1)^3$ case reads

$$4\kappa_1 c_1(L_1) + (\kappa_1 - 5\kappa_2)\, c_1(L_2) + (6\kappa_3 + \kappa_1 - \kappa_2)\, c_1(L_3) = 0. \tag{127}$$

For linearly independent first Chern classes, the respective equation cannot be satisfied other than trivially, of course, and we are left with gauge group $SU(3)\times SU(2)$. If, however, the $c_1(L_i)$ span a two- or one-dimensional subspace of their cohomology class, we can find – modulo rescaling – precisely one or, respectively, two non-anomalous $U(1)_f$. These $U(1)$ symmetries are of type (ii) and, as they cannot receive a mass via GS-couplings, remain massless.

Independently of the concrete bundle data, one can check that quite a few values of $\kappa_1, \kappa_2, \kappa_3$ admit an interpretation of the corresponding abelian factor, if massless, as the MSSM hypercharge $U(1)_Y$. We list them in Table 7 and Table 8 together with the respective candidates for MSSM fermions exhibiting the required $SU(3) \times SU(2) \times U(1)_Y$ quantum numbers.

A closer look at Table 10 reveals a large number of possibilities for further non-abelian gauge enhancements for those choices of $c_1(L_1), c_1(L_2), c_1(L_3)$ where additional gauge bosons in the $H^*(\mathcal{M}, \mathcal{O})$ representation arise. In fact, one can verify that the spectrum then organises itself into multiplets of the corresponding

part.	class	$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{10} \\ -\frac{1}{15} \end{pmatrix}$	$\begin{pmatrix} -\frac{5}{14} \\ \frac{1}{14} \\ -\frac{13}{21} \end{pmatrix}$	$\begin{pmatrix} \frac{3}{2} \\ -\frac{1}{10} \\ \frac{1}{15} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} \\ \frac{33}{30} \\ -\frac{1}{15} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{10} \\ -\frac{7}{15} \end{pmatrix}$
Q_L	D	$1,2,4$	$1,3$	1	$2,3$	4	4
$\overline{U}_R$	C	$2,3,4$	$4,6$	$6,7$	$4,7$	$4,7$	$4,6$
$\overline{D}_R$	C	$1,5,6,7$	2	1	3	$1,2,5$	$1,3,5$
L	B	$1,2,3,\overline{4}$	3	$\overline{4}$	2	$1,\overline{3},\overline{4}$	$1,\overline{2},\overline{4}$
$\overline{E}_R$	A	$\overline{2},\overline{3},6$	$\overline{4},\overline{6}$	$\overline{4},\overline{5}$	$\overline{5},\overline{6}$	$\overline{4},5,6$	$\overline{4},5,6$
$\overline{\nu}_R$	A	$1,4,5$	2	1	3	3	1

Table 7: MSSM particle candidates for choices of $(\kappa_1, \kappa_2, \kappa_3)$, part I. The labels of the representations refer to the position in the respective sections of Table 10 with bars denoting hermitian conjugation.

gauge group, as listed in Table 9. We arrive at even higher rank gauge groups if several of the states transform in the trivial bundle simultaneously. The resulting enhancement pattern is plotted schematically in Figure 4 for the case that V has structure group $SU(3)$. An analogous pattern can of course be derived for the $U(3)$ bundle construction.

5.2 Example: A model with Standard Model gauge symmetry

In this section we present one example our simple survey of $U(3) \times U(1)^3$ vector bundles revealed, which shows that indeed the framework is rich enough to contain models with just the Standard Model gauge symmetry (in addition to a hidden gauge symmetry from the second E_8 factor). We consider the following Calabi-Yau

$$\mathcal{M} = \begin{array}{c} \mathbb{P}_2 \\ \mathbb{P}_1 \\ \mathbb{P}_1 \end{array} \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} \tag{128}$$

with Hodge numbers $(h_{21}, h_{11}) = (75, 3)$. Let η_1 denote the two-form on $\mathbb{P}_2$ and η_2, η_3 the two-forms on the two $\mathbb{P}_1$ spaces. The resulting Stanley-Reisner ideal $SR = \{\eta_1^3, \eta_2^2, \eta_3^2\}$ on the ambient space eventually determines the intersection form on the Calabi-Yau

$$I_3 = 3\,\eta_1\eta_2\eta_3 + 2\,\eta_1^2\eta_2 + 2\,\eta_1^2\eta_3. \tag{129}$$

part.	class	$\begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{4} \\ \frac{3}{20} \\ -\frac{4}{15} \end{pmatrix}$	$\begin{pmatrix} -1 \\ \frac{1}{5} \\ -\frac{7}{15} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{12} \\ \frac{7}{60} \\ -\frac{1}{15} \end{pmatrix}$	$\begin{pmatrix} -1 \\ \frac{3}{5} \\ -\frac{1}{15} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2} \\ \frac{7}{10} \\ -\frac{7}{15} \end{pmatrix}$
Q_L	D	4	$1,3$	1	$2,3$	2	3
$\overline{U}_R$	C	$6,7$	5	6	5	7	4
$\overline{D}_R$	C	$2,3,5$	$2,7$	$4,7$	$3,6$	$6,4$	$6,7$
L	B	$1,\overline{2},\overline{3}$	$\overline{2},\overline{4}$	$\overline{3},\overline{4}$	$\overline{3},\overline{4}$	$\overline{2},\overline{4}$	$\overline{2},\overline{3}$
$\overline{E}_R$	A	$4,5,\overline{6}$	$\overline{5}$	$\overline{1},2,\overline{4},\overline{5}$	$\overline{4}$	$1,3,\overline{5}$	$\overline{2},\overline{3},\overline{6}$
$\overline{\nu}_R$	A	1	2	3	3	2	1

Table 8: MSSM particle candidates for choices of $(\kappa_1, \kappa_2, \kappa_3)$, part II.

Due to $\eta_2^2 = \eta_3^2 = 0$ there are naively four four-forms on $\mathcal{M}$, namely $\{\eta_1^2, \eta_1\eta_2, \eta_1\eta_3, \eta_2\eta_3\}$, related, however, via

$$9\,\eta_1^2 - 6\,\eta_1\eta_2 - 6\,\eta_1\eta_3 + 8\,\eta_2\eta_3 = 0. \tag{130}$$

Invoking the relation (130) to eliminate $\eta_1\eta_3$, the second Chern class of the manifold can be written as

$$c_2(T) = 12\,\eta_1^2 + 12\,\eta_2\eta_3. \tag{131}$$

Now we choose the three line bundles on $\mathcal{M}$

$$L_1 = \mathcal{O}(-1,1,0), \qquad L_2 = \mathcal{O}(-1,0,1), \qquad L_3 = \mathcal{O}(0,-1,1), \tag{132}$$

satisfying $c_1(L_2) = c_1(L_1) + c_1(L_3)$ and no other linear relation. Looking back at (125) and (127), one realizes that this is consistent with the linear combination

$$U(1) = \frac{1}{2}\,U(1)_Z + \frac{1}{2}\,U(1)_X + \frac{1}{3}\,U(1)_{Y'} \tag{133}$$

being anomaly-free. Moreover, from Table 7 we learn that under this $U(1)$ gauge symmetry some chiral fields have Standard Model quantum numbers. Therefore, this $U(1)$ defines a possible hypercharge $U(1)_Y$. Since there is no further relation among the $c_1(L_i)$, the remaining two $U(1)$s receive a mass via the Green-Schwarz mechanism and the $U(1)_Y$ as a type (ii) gauge factor stays massless.

Now we define a $U(3)$ bundle with $c_1(V) = \sum_i c_1(L_i)$ by the exact sequence

$$0 \to\; V \to \mathcal{O}(0,1,0) \oplus \mathcal{O}(0,0,1) \oplus \mathcal{O}(0,0,2) \oplus \mathcal{O}(1,1,1)|_{\mathcal{M}} \xrightarrow{f} \mathcal{O}(3,2,2)|_{\mathcal{M}} \to 0. \tag{134}$$

	rep.	$SU(3)\times U(1)^3$	$U(3)\times U(1)^3$	gauge group
A1	$(1,1,1)_{0,4,6}$	$2l_2+3l_3=0$	$l_2-l_3=0$	$SU(3)\times SU(2)^2$
A2	$(1,1,1)_{-3,-5,0}$	$3l_1+5l_2=0$	$l_1-l_2=0$	$SU(3)\times SU(2)^2$
A3	$(1,1,1)_{-3,-1,6}$	$3l_1+l_2-6l_3=0$	$l_1-l_3=0$	$SU(3)\times SU(2)^2$
B1	$(1,1,\mathbf{2})_{-3,3,-3}$	$l_1-l_2+l_3=0$	$l_1+l_2+l_3=0$	$SU(3)\times SU(3)$
C1	$(1,\overline{\mathbf{3}},1)_{0,4,-4}$	$l_2-l_3=0$	$l_2+l_3=0$	$SU(4)\times SU(2)$
C2	$(1,\overline{\mathbf{3}},1)_{-3,3,-2}$	$3l_1-2l_2+3l_3=0$	$l_1+l_2=0$	$SU(4)\times SU(2)$
C3	$(1,\overline{\mathbf{3}},1)_{-3,-1,-4}$	$3l_1+l_2+4l_3=0$	$l_1+l_3=0$	$SU(4)\times SU(2)$
D1	$(1,\mathbf{3},\mathbf{2})_{0,4,1}$	$4l_2+l_3=0$	$l_2=0$	$SU(5)$
D2	$(1,\mathbf{3},\mathbf{2})_{0,0,-5}$	$l_3=0$	$l_3=0$	$SU(5)$
D3	$(1,\mathbf{3},\mathbf{2})_{-3,-1,1}$	$3l_1+l_2-l_3=0$	$l_1=0$	$SU(5)$

Table 9: Generic enhancement of $SU(3)\times SU(2)$ by additional non-chiral degrees of freedom for both the $SU(3)\times U(1)^3$ and $U(3)\times U(1)^3$ case. We use the notation $l_i=c_1(L_i)$.

Computing the second Chern classes one obtains

$$-\,\mathrm{ch}_2(V)-\frac{1}{2}\sum_{i=1}^{3}c_1^2(L_i)=12\,\eta_1^2+12\,\eta_2\eta_3=c_2(T), \tag{135}$$

which means that the tadpole cancellation condition is fulfilled right away.

It is now straightforward to compute the chiral massless spectrum for this model. The result is shown in Appendix C and exemplifies that particles with Standard Model like quantum numbers appear, but in addition there are some exotic chiral fields with fractional electric charges. In this example the chiral exotics imply that the Standard Model particles by themselves do not render the gauge theory anomaly-free. However, one realizes that “accidentally” many chiral exotics are absent, which we think is quite encouraging for realistic string model building. One could also determine the non-chiral matter by computing the various cohomology groups. We find for instance that the complete quark-dublett spectrum reads

$$H^*(\mathcal{M},V)=(0,70,1,0) \tag{136}$$

so that there is also one anti-generation. Since this model is not fully realistic anyway we skip this elaborate computation here.

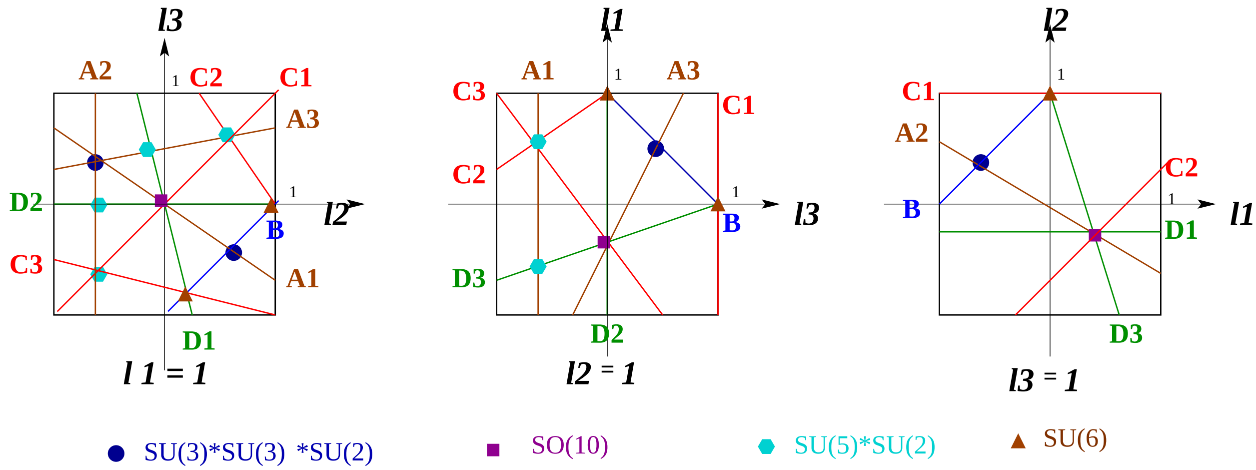

Figure 4: Gauge symmetry enhancement for $SU(3)\times U(1)^3$ bundles. The picture shows the projection of the various planes defined in Table 9 into the planes $l_i \equiv c_1(L_i) = 1$. At the point $l_i = 0$ for $i = 1, 2, 3$, the observable gauge group is E_6.

Instead, we conclude that, as for intersecting D-brane models, extra constraints have to be imposed to guarantee the absence of chiral exotics. In general of course there can appear also non-chiral exotics, which are visible by computing the precise cohomology classes of all the bundles involved.

Finally let us analyse the implications of the supersymmetry condition. With the ansatz $J = \ell_s^2 \sum_i r_i \eta_i$ the DUY condition reads for a general line bundle $\mathcal{O}(m, n, l)$

$$r_1 r_2 (3l + 2m) + r_1 r_3 (3n + 2m) + r_2 r_3 (3m) + r_1^2 (n + l) = \tag{137}$$

$$e^{2\phi_{10}} \, (-\frac{9}{2} m - 3n - 3l). \tag{138}$$

Our choice of bundles results in two linearly independent equations, one of which,

$$r_1 \, (r_2 - r_3) = 0, \tag{139}$$

dictates that we need to satisfy $r_3 = r_2$ for stabilization inside the Kähler cone. In addition, the dilaton is related to the two other radii of the Calabi-Yau via

$$r_1^2 - 3\, r_2^2 - r_1\, r_2 = \frac{3}{2}\, e^{2\phi_{10}}, \tag{140}$$

which clearly admits a solution with $r_1 > 0, r_2 > 0$ for every fixed value of the dilaton. The solution to this equation for various values of the string coupling constant is shown in Figure 5.

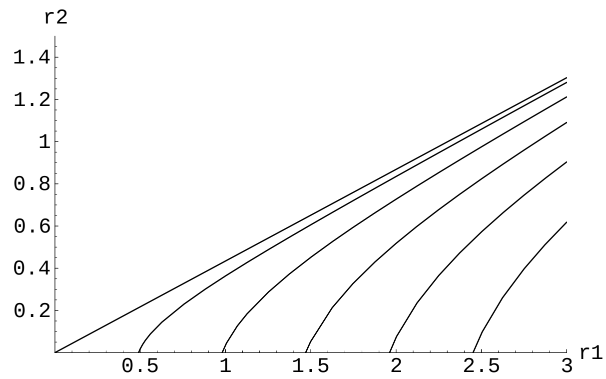

Figure 5: The plot shows the Kähler moduli (r_1, r_2) for the values $4g_s \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$ of the string coupling constant.

6 Conclusions

In this article we have investigated the model building prospects of embedding bundles with $U(1)$ factors in their structure group into the $E_8 \times E_8$ gauge bundle of the heterotic string. The generic features occurring are very similar to the Type I side. One encounters anomalies for multiple $U(1)$ gauge factors (of type (i)) which are canceled by a generalised Green-Schwarz mechanism involving both the axio-dilaton and the axio-Kähler multiplets. The induced masses for the $U(1)$ gauge bosons can in principle be anywhere between the weak and the string scale. Moreover, the accompanying FI-terms contain the tree-level Donaldson-Uhlenbeck-Yau equation in addition to a one-loop correction, which has just the right form to be consistent with Type I-heterotic string duality. The loop-corrected DUY condition freezes combinations of the dilaton and the Kähler moduli. Again similar to Type I strings, the gauge couplings for the $U(1)$ gauge factors of type (i) are non-universal if one includes the one-loop threshold corrections.

Concerning concrete model building, we have demonstrated that it is possible to break the ten-dimensional gauge symmetry directly to the Standard Model gauge group. For special combinations of line bundles the models experience non-abelian gauge symmetry enhancements, which for many $U(1)$ factors show a rich pattern of possible gauge groups. As on the Type I side, new (chiral) exotic fields with non-Standard Model quantum numbers do arise, whose absence imposes extra constraints on the respective bundle cohomology classes.

Using exact sequences of sums of line bundles, we have also constructed a number of concrete examples showing that indeed bundles of the described type can be found. The models presented here are not yet realistic, but we do not see any conceptual obstacle to obtaining more realistic heterotic string models of this type. Of course, for concreteness we studied just a very few possible

embeddings of $U(1)$ structure groups into $E_8 \times E_8$. As with intersecting D-branes, many more embeddings are possible and some of them might turn out to be of phenomenological interest.

From the phenomenological point of view, there are a couple of open questions. We have not addressed Yukawa couplings in this paper and it would be interesting to see whether they can give rise to a hierarchy of fermion masses. Moreover, one could try to freeze some of the complex structure and bundle moduli by turning on some background H-flux [20,56,57]. As on the Type IIB side, a supersymmetry breaking flux is expected to induce soft terms in the observable gauge sector. The analogous story for the $SO(32)$ heterotic string will be slightly different, as there exists an independent fourth order Casimir. We plan to address some of these issues in [54].

On the more formal side, the question arises whether these models also admit a linear sigma model [6] respectively Landau-Ginzburg [58] or conformal field theory description [59]. This would be interesting, as it is by now well established that such models are not destabilized by world-sheet instantons [60–62]. Moreover, one could look for perturbative dualities among such models like $(0,2)$ mirror symmetry [63–66] or the dualities described in [67,68].

Last but not least, these heterotic string vacua constitute an additional branch in the string theory landscape and one might try to invoke statistical methods to get estimates on the distributions of various physical quantities in the gauge theory sector [69–72].

Acknowledgments

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A Computing cohomology classes

In general a short exact sequence of sheaves

$$0 \to A \xrightarrow{\alpha} B \xrightarrow{\beta} C \to 0 \tag{141}$$

implies a long exact sequence in cohomology

$$0 \to H^0(M,A) \xrightarrow{\alpha} H^0(M,B) \xrightarrow{\beta} H^0(M,C) \xrightarrow{\phi} H^1(M,A) \xrightarrow{\alpha} H^1(M,B) \to \ldots \tag{142}$$

The maps α and β in (142) are induced from the sheaf homomorphisms in (141). For the definition of ϕ we refer to the mathematical literature, but it is emphasised that the definition of ϕ relies on the shortness of the sequence (141).

In order to use the long exact cohomological sequences (142) in our case, one has to know the cohomology classes of line bundles restricted to the complete intersection locus defining the Calabi Yau 3-fold. To this end one uses the Koszul sequence for a complete intersection of K hypersurfaces $\xi = (f_1, \ldots, f_K)$ with f_i a section of the line bundle $\mathcal{E}_{f_i}$ over the ambient space

$$0 \to \wedge^K \mathcal{E}^* \xrightarrow{\xi\cdot} \ldots \xrightarrow{\xi\cdot} \wedge^2 \mathcal{E}^* \xrightarrow{\xi\cdot} \mathcal{E}^* \xrightarrow{\xi\cdot} \mathcal{O} \xrightarrow{\rho\cdot} \mathcal{O}|_{\mathcal{M}} \to 0. \tag{143}$$

Here $\mathcal{E} = \bigoplus \mathcal{E}_{f_i}$, $\mathcal{O}$ denotes the structure sheaf of the ambient space and ρ is the restriction map.

For computing the cohomology of anti-symmetric products of bundles, a useful fact is that a short exact sequence (141) implies the following set of exact sequences[9]

$$\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
0 & \to & \bigwedge^2 A & \to & Q_1 & \to & A \otimes C & \to & 0 \\
 & & & & \downarrow & & \downarrow & & \\
0 & \to & \bigwedge^2 A & \to & \bigwedge^2 B & \to & Q_2 & \to & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & \bigwedge^2 C & & \bigwedge^2 C & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array} \tag{144}$$

which for C being a line bundle reduces to the following short exact sequence

$$0 \to \bigwedge^2 A \to \bigwedge^2 B \to A \otimes C \to 0. \tag{145}$$

B The general massless spectrum for $SU(3) \times U(1)^3$ bundles

The massless spectrum is counted by the cohomology classes in Table 10.

C The massless spectrum for the $SU(3) \times SU(2) \times U(1)_Y$ example

In Table 11 we list the chiral massless spectrum of the model discussed in section 5.2 with Standard Model gauge symmetry.

[9]We thank Volker Braun for communicating this mathematical fact to us.

class	reps.	$SU(3)\times U(1)^3$	$U(3)\times U(1)^3$
$D1$	$(\mathbf{1};\mathbf{3},\mathbf{2})_{0,4,1}$	$H^*(\mathcal{M},L_2^4\otimes L_3^1)$	$H^*(\mathcal{M},L_2^{-1})$
$D2$	$(\mathbf{1};\mathbf{3},\mathbf{2})_{0,0,-5}$	$H^*(\mathcal{M},L_3^{-5})$	$H^*(\mathcal{M},L_3^{-1})$
$D3$	$(\mathbf{1};\mathbf{3},\mathbf{2})_{-3,-1,1}$	$H^*(\mathcal{M},L_1^{-3}\otimes L_2^{-1}\otimes L_3^1)$	$H^*(\mathcal{M},L_1^{-1})$
$D4$	$(\mathbf{3};\mathbf{3},\mathbf{2})_{1,-1,1}$	$H^*(\mathcal{M},V\otimes L_1^1\otimes L_2^{-1}\otimes L_3^1)$	$H^*(\mathcal{M},V)$
$B1$	$(\mathbf{1};\mathbf{1},\mathbf{2})_{-3,3,-3}$	$H^*(\mathcal{M},L_1^{-3}\otimes L_2^3\otimes L_3^{-3})$	$H^*(\mathcal{M},L_1^{-1}\otimes L_2^{-1}\otimes L_3^{-1})$
$B2$	$(\mathbf{3};\mathbf{1},\mathbf{2})_{-2,-2,-3}$	$H^*(\mathcal{M},V\otimes L_1^{-2}\otimes L_2^{-2}\otimes L_3^{-3})$	$H^*(\mathcal{M},V\otimes L_1^{-1}\otimes L_3^{-1})$
$B3$	$(\mathbf{3};\mathbf{1},\mathbf{2})_{-2,2,3}$	$H^*(\mathcal{M},V\otimes L_1^{-2}\otimes L_2^2\otimes L_3^3)$	$H^*(\mathcal{M},V\otimes L_1^{-1}\otimes L_2^{-1})$
$B4$	$(\mathbf{3};\mathbf{1},\mathbf{2})_{1,3,-3}$	$H^*(\mathcal{M},V\otimes L_1^1\otimes L_2^3\otimes L_3^{-3})$	$H^*(\mathcal{M},V\otimes L_2^{-1}\otimes L_3^{-1})$
$C1$	$(\mathbf{1};\overline{\mathbf{3}},\mathbf{1})_{0,4,-4}$	$H^*(\mathcal{M},L_2^4\otimes L_3^{-4})$	$H^*(\mathcal{M},L_2^{-1}\otimes L_3^{-1})$
$C2$	$(\mathbf{1};\overline{\mathbf{3}},\mathbf{1})_{-3,3,2}$	$H^*(\mathcal{M},L_1^{-3}\otimes L_2^3\otimes L_3^2)$	$H^*(\mathcal{M},L_1^{-1}\otimes L_2^{-1})$
$C3$	$(\mathbf{1};\overline{\mathbf{3}},\mathbf{1})_{-3,-1,-4}$	$H^*(\mathcal{M},L_1^{-3}\otimes L_2^{-1}\otimes L_3^{-4})$	$H^*(\mathcal{M},L_1^{-1}\otimes L_3^{-1})$
$C4$	$(\mathbf{3};\overline{\mathbf{3}},\mathbf{1})_{-2,-2,2}$	$H^*(\mathcal{M},V\otimes L_1^{-2}\otimes L_2^{-2}\otimes L_3^2)$	$H^*(\mathcal{M},V\otimes L_1^{-1})$
$C5$	$(\overline{\mathbf{3}};\overline{\mathbf{3}},\mathbf{1})_{2,-2,2}$	$H^*(\mathcal{M},\bigwedge^2 V\otimes L_1^2\otimes L_2^{-2}\otimes L_3^2)$	$H^*(\mathcal{M},\bigwedge^2 V)$
$C6$	$(\mathbf{3};\overline{\mathbf{3}},\mathbf{1})_{1,3,2}$	$H^*(\mathcal{M},V\otimes L_1^1\otimes L_2^3\otimes L_3^2)$	$H^*(\mathcal{M},V\otimes L_2^{-1})$
$C7$	$(\mathbf{3};\overline{\mathbf{3}},\mathbf{1})_{1,-1,-4}$	$H^*(\mathcal{M},V\otimes L_1^1\otimes L_2^{-1}\otimes L_3^{-4})$	$H^*(\mathcal{M},V\otimes L_3^{-1})$
$A1$	$(\mathbf{1};\mathbf{1},\mathbf{1})_{0,4,6}$	$H^*(\mathcal{M},L_2^4\otimes L_3^6)$	$H^*(\mathcal{M},L_2^{-1}\otimes L_3)$
$A2$	$(\mathbf{1};\mathbf{1},\mathbf{1})_{-3,-5,0}$	$H^*(\mathcal{M},L_1^{-3}\otimes L_2^{-5})$	$H^*(\mathcal{M},L_1^{-1}\otimes L_2)$
$A3$	$(\mathbf{1};\mathbf{1},\mathbf{1})_{-3,-1,6}$	$H^*(\mathcal{M},L_1^{-3}\otimes L_2^{-1}\otimes L_3^6)$	$H^*(\mathcal{M},L_1^{-1}\otimes L_3)$
$A4$	$(\mathbf{3};\mathbf{1},\mathbf{1})_{1,-5,0}$	$H^*(\mathcal{M},V\otimes L_1^1\otimes L_2^{-5})$	$H^*(\mathcal{M},V\otimes L_2)$
$A5$	$(\mathbf{3};\mathbf{1},\mathbf{1})_{1,-1,6}$	$H^*(\mathcal{M},V\otimes L_1^1\otimes L_2^{-1}\otimes L_3^6)$	$H^*(\mathcal{M},V\otimes L_3)$
$A6$	$(\mathbf{3};\mathbf{1},\mathbf{1})_{4,0,0}$	$H^*(\mathcal{M},V\otimes L_1^4)$	$H^*(\mathcal{M},V\otimes L_1)$

Table 10: Massless spectrum of $H=SU(3)\times SU(2)\times U(1)^3$ models.

part.	reps.	class	χ	χ_{tot}	Q_Y
Q_L	$(\mathbf{3},\mathbf{2})_{1,-1,1}$	$D4$	-69	-69	$\frac{1}{3}$
U_R	$(\overline{\mathbf{3}},\mathbf{1})_{-2,-2,2}$	$C4$	-75		$-\frac{4}{3}$
U_R	$(\overline{\mathbf{3}},\mathbf{1})_{1,-1,4}$	$C7$	-63	-138	$-\frac{4}{3}$
D_R	$(\overline{\mathbf{3}},\mathbf{1})_{0,4,-4}$	$C1$	-6		$\frac{2}{3}$
D_R	$(\overline{\mathbf{3}},\mathbf{1})_{-3,3,2}$	$C2$	0		$\frac{2}{3}$
D_R	$(\overline{\mathbf{3}},\mathbf{1})_{2,-2,2}$	$C5$	75	69	$\frac{2}{3}$
L_L	$(\mathbf{1},\mathbf{2})_{-3,3,-3}$	$B1$	-6		-1
L_L	$(\mathbf{1},\mathbf{2})_{2,-2,-3}$	$\overline{B3}$	75		-1
L_L	$(\mathbf{1},\mathbf{2})_{-1,-3,3}$	$\overline{B4}$	69	138	-1
E_R	$(\mathbf{1},\mathbf{1})_{-1,5,0}$	$\overline{A4}$	57		2
E_R	$(\mathbf{1},\mathbf{1})_{1,-1,6}$	$A5$	-63		2
E_R	$(\mathbf{1},\mathbf{1})_{4,0,0}$	$A6$	-63	-69	2
N_R	$(\mathbf{1},\mathbf{1})_{-3,-1,6}$	$A3$	-6	-6	0
Q_L^{ex}	$(\mathbf{3},\mathbf{2})_{0,4,1}$	$D1$	0		$\frac{7}{3}$
Q_L^{ex}	$(\mathbf{3},\mathbf{2})_{0,0,-5}$	$D2$	0		$-\frac{5}{3}$
Q_L^{ex}	$(\mathbf{3},\mathbf{2})_{-3,-1,1}$	$D3$	0	0	$-\frac{5}{3}$
Q_R^{ex}	$(\overline{\mathbf{3}},\mathbf{1})_{-3,-1,-4}$	$C3$	0		$-\frac{10}{3}$
Q_R^{ex}	$(\overline{\mathbf{3}},\mathbf{1})_{1,3,2}$	$C6$	-69	-69	$\frac{8}{3}$
L_L^{ex}	$(\mathbf{1},\mathbf{2})_{-2,-2,-3}$	$B2$	-69	-69	-3
L_R^{ex}	$(\mathbf{1},\mathbf{1})_{0,4,6}$	$A1$	0		4
L_R^{ex}	$(\mathbf{1},\mathbf{1})_{3,5,0}$	$\overline{A2}$	0	0	4

Table 11: Massless spectrum of $H = SU(3) \times SU(2) \times U(1)_Y$ model.

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